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# EXPERIMENTÁLNY VÝSKUM PEVNOSTI ANIZOTROPNÝCH TELIES PRI DVOJOSOVOM NAMÁHANÍ ŠIKMO K HLAVNÝM SMEROM MATERIÁLU

# EXPERIMENTAL INVESTIGATION OF THE STRENGTH OF ANISOTROPIC SOLIDS UNDER BIAXIAL LOADING OBLIQUE TO THE PRINCIPAL MATERIAL DIRECTIONS

*Realistická konečno-prvková analýza medzného zaťaženia konštrukčných častí ako aj škrupinových konštrukcií vyrobených z anizotropných materiálov vyžaduje vhodné konštitutívne rovnice na predpoveď chovania sa deformácie a medznej pevnosti dvojosovo zaťažených telies. Avšak je nedostatok vhodných biaxiálnych experimentálnych údajov, zvlášť v situáciách, keď sa uvažuje, že hlavné smery zaťaženia nie sú totožné s hlavnými smermi materiálových vlastností.*

*Z teoretického hľadiska musí byť splnený vzťah napätie-deformácia. Nie je zásadný rozdiel v zohľadňovaní rôznych druhov anizotropných materiálov, ako sú biologické tkanivá (ľudská koža), tkanivá (textil), vystužené polyméry a iné druhy plasticky deformovaných membrán. Avšak návrh experimentu, konštrukcia zaťažovacieho zariadenia a postup zaťažovania sú silno závislé od tvaru a veľkosti danej vzorky (alebo vzhľadom na potreby výskumu) a od mechanických vlastností uvažovaného materiálu.*

*V tomto článku je ukázaný vývoj vhodného skúšobného zariadenia ako aj skúšobných vzoriek pre prípad ortotropického dreva. Cieľom výskumu je určenie pevnosti materiálu a dvojosového vzťahu napätie-deformácia dreva, keď je zaťažované šikmo k štruktúre materiálu.*

*Realistic finite element ultimate load analyses of structural details as well as of shell structures made of anisotropic materials require suitable constitutive equations for the prediction of the deformational behavior and ultimate strength of biaxially loaded solids. However, there is a lack of adequate biaxial experimental data, particularly if loading situations are considered where the principal loading directions do not coincide with the principal material directions.*

*From the theoretical point of view one has to obey the stress-strain relationship. There is no principal difference in dealing with different kinds of anisotropic materials, such as biological tissues (human skin), anisotropic rubbers, woven fabrics (textures), reinforced polymers and any kind of plastic deformed membranes. However, the design of the experiment, the construction of the loading device and the loading procedure are strongly influenced by the shape and size of the test specimen available (or in view to the goal of the investigation necessary) and by the mechanical properties of the material under consideration.*

*In this paper the development of an adequate testing device as well as a testing specimen is shown for the case of orthotropic wood. The goals of this investigation are the determination of the material strength and the biaxial stress-strain relationship of wood when loaded oblique to the grain direction.*

## 1. Mechanical Fundamentals

For the description of the mechanical properties of an anisotropic material it is suitable to use the generalized Hooke's Law. However, in all cases with large deformations we have to be aware that the stress-strain relationship up to failure could be non-linear. In such cases the linearized descriptions are usually restricted to small deformation steps (see Fig. 1).

Therefore in such an investigation a stepwise loading procedure is used as illustrated in Fig. 2 [1]. In case of wood it has been shown in an experimental study that the rheological effects may be neglected if the holding time does not exceed a period of about 15 seconds.

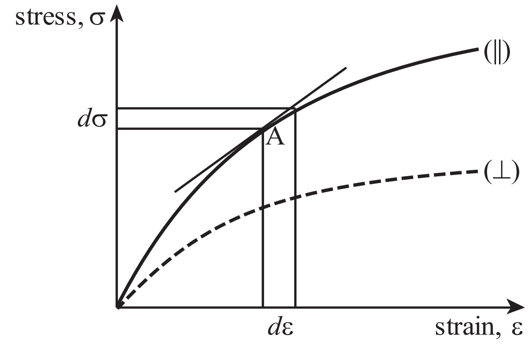

*Fig. 1 Stress-strain relation for an anisotropic material with large deformations*

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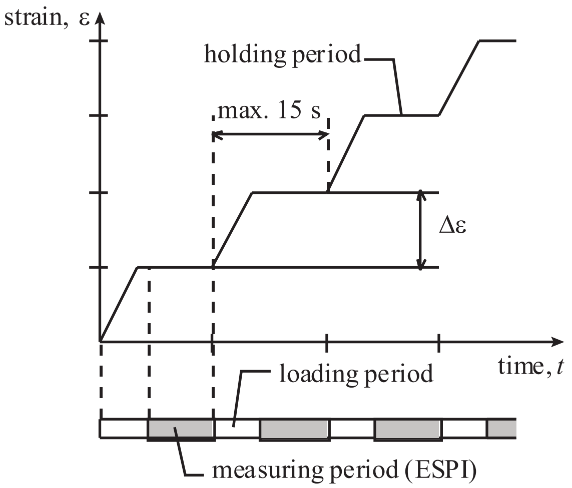


*Fig. 2 Stepwise loading procedure*

The final deformation is the result of the sum of the small loading steps and the experimental procedure meets the conditions of additivity according to Hencky's strain definition [2]:

$$d\varepsilon_H = \frac{dl}{l}\,;\;\; \varepsilon_H = \int_{l_0}^{l} d\varepsilon_H \tag{1.1}$$

$$\begin{aligned}\varepsilon_H = \varepsilon_{H1} + \varepsilon_{H2} &= \ln\frac{l_0+\Delta l_1}{l_0} + \ln\frac{l_0+\Delta l_1+\Delta l_2}{l_0+\Delta l_1} = \\ &= \ln\frac{l_0+\Delta l_1+\Delta l_2}{l_0} = \varepsilon_{H,\,1+2}\end{aligned} \tag{1.2}$$

With the restriction to plane stress loading of thin test specimens the remaining generalized Hooke's Law is given by equations (2.1) - (2.4)

$$d\varepsilon_x = a_{11}d\sigma_x + a_{12}d\sigma_y + a_{16}d\tau_{xy} \tag{2.1}$$

$$d\varepsilon_y = a_{21}d\sigma_x + a_{22}d\sigma_y + a_{26}d\tau_{xy} \tag{2.2}$$

$$d\varepsilon_z = a_{31}d\sigma_x + a_{32}d\sigma_y + a_{36}d\tau_{xy} \tag{2.3}$$

$$d\gamma_{xy} = a_{61}d\sigma_x + a_{62}d\sigma_y + a_{66}d\tau_{xy} \tag{2.4}$$

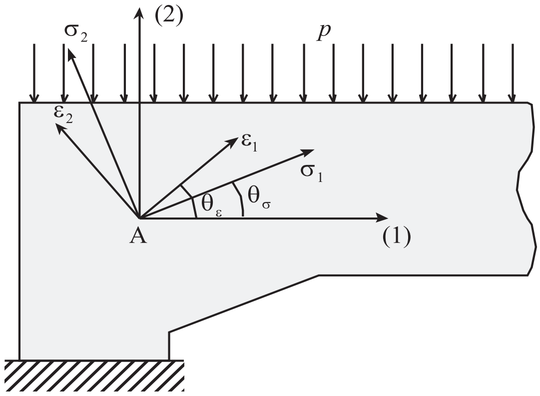


*Fig. 3 Example for an investigation of a wooden console*

In cases where a numerical investigation of a flat anisotropic structure shows a state of biaxial stress and strain (x-y plane) with non-coincident principal directions (Fig. 3) usually no information about failure can be found in the literature. In order to eliminate this deficiency, a comprehensive experimental investigation of the stiffness and strength behavior of the material under arbitrary two-dimensional loading conditions was carried out in advance for which an adequate method had to be developed.

The experimental parameters of such biaxial investigations are the ratio between the applied principal stress components and the angle between principal loading direction and principal material direction $\Theta_\sigma$ (Fig. 4). The design of an adequate testing procedure (loading device as well as test specimen) is strongly influenced by the material under investigation, by shape and size of the test specimen available and of course by the goal of investigation.

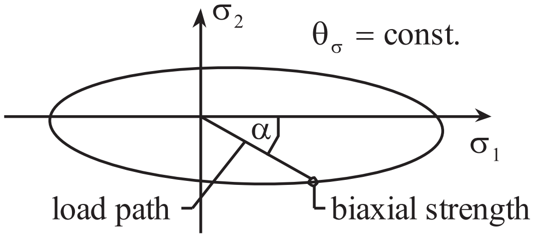


*Fig. 4 Failure envelope*

The aim of this experimental study is to provide information about the ultimate strength behavior under biaxial loading oblique to the grain direction of wood. Due to the fact that the specimen's surfaces ($z = \pm\, t\,/\,2$) are stress-free the strains in z-direction, concerning Eqn. (2.3), will be neglected in the following considerations. From this it follows that only 6 coefficients of the generalized Hooke's Law (2.1) - (2.4) are essential.

## 2. Experimental Setup

With the viewpoint of the special task of this investigation the test specimen of finite size has to simulate an infinite element of the material assumed in a numerical calculation. That leads to some essential requirements [3]: first, a homogeneous testing field (humidity, density, grain direction and grain density) and, second, failure due to the applied homogeneous stress and strain distributions should occur within the region of the testing field. Beside a very careful selection and storage of the material it is necessary to optimize the loading procedure, which is most important for the construction for both, the loading device as well as the test specimen developed by the authors. The final shape of the test specimen is shown in Fig. 5 and the loading device is shown in Fig. 6.

In order to achieve homogeneous states of stress and strain with this equipment a certain strategy of loading was necessary, which is described by Eqns. (3.1) - (3.3):

$$d\varepsilon_x = a_{11}d\sigma_x + a_{12}d\sigma_y \tag{3.1}$$

$$d\varepsilon_y = a_{21}d\sigma_x + a_{22}d\sigma_y \tag{3.2}$$

$$d\gamma_{xy} = a_{61}d\sigma_x + a_{62}d\sigma_y \tag{3.3}$$

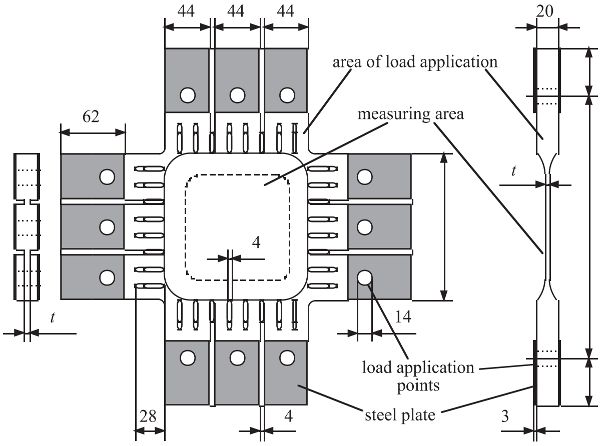
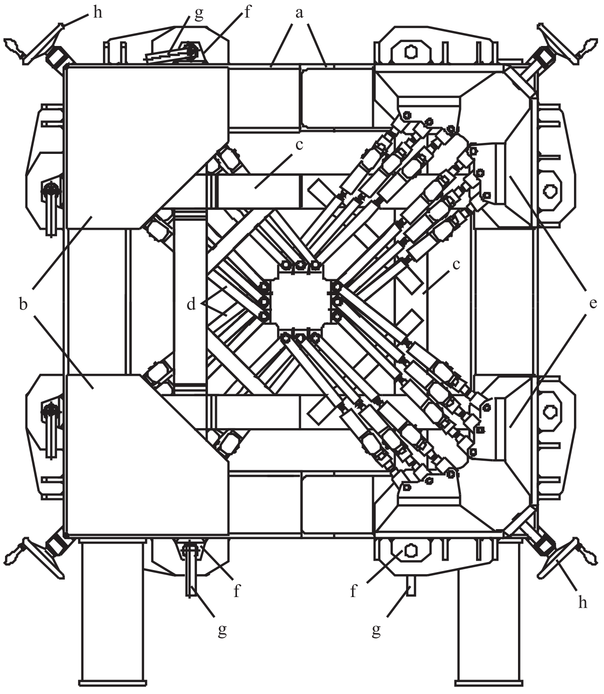


*Fig. 5 Test specimen*



a......double steel frame
b......stiffening plate
c......stiffening frame
d......loading axes
e......ancorage unit
f......plug holding device
g......connecting plug
h......crank handle

*Fig. 6 Hydraulic loading device*

That means that the test specimen was loaded in such a way that shear stresses are vanishing in the principal directions of the testing device. A FE study showed the necessary movement of the loading points in order to obtain this specific state of stress for any angle between the principal directions of the stresses an the grain direction of wood within the testing field. In this way it was also possible to avoid unwanted stress and strain concentrations at the corners of the test specimen.

The very specific properties of wood require a contactless measuring method for the deformation analysis of the specimen [4]. Beyond that the chosen method must be at least two-dimensional, full-field accurate and in real-time. Therefore, from the viewpoint of the enormous amount of necessary experiments from all available optical methods the Electronic Speckle-Pattern Interfe-rometry (ESPI) is the only practicable one.

The developed ESPI system, described in detail in [1], is mounted at the hydraulic loading device through two independent frame sets in order to avoid vibrations to the optical set-up. The measuring system allows a full-field analysis of two in-plane components and one out-of-plane component of the state of deformation in the measuring region (140 x 140 mm) of a biaxial specimen (Fig. 5).

## 3. Experimental Results

The displacement-controlled experiments were characterized by a proportional stepwise loading until fracture was reached. The applied displacement steps varied from 4 to 10 μm. The experimentally obtained failure envelopes for the investigated grain directions $\varphi = 0°$, 7.5°, 15°, 30° and 45° reflect the strong influence of grain direction on the biaxial strength behavior of solid wood as it is known from uniaxial strength tests. The obtained experimental results form an essential basis for further developments in material modelling. In Figs. 7 to 10 representative experimental results are shown.

Fig. 7 contains the stepwise progression of load application forces for an experimental configuration characterized by a grain angle of $\varphi = 15°$ and a displacement ratio of $\kappa = +1{:}\ -2$. The latter results in a tensile loading in horizontal direction and in a compressive loading in vertical direction. The shown results

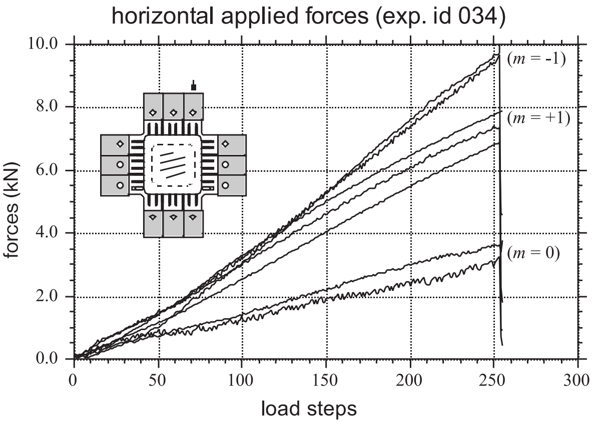


*Fig. 7 Load application forces resulting from displacement ratio $\kappa = +1{:}-2$*

indicate that the forces at the middle load application point ($m = 0$) are much lower than the ones at the outer load application points ($m = \pm 1$).

The biaxial strength values for a representative number of displacement ratios and a grain direction of $\varphi = 30°$ are shown in Fig. 8. For each displacement ratio six experiments were carried out. The obtained fracture points in the biaxial stress space are in good correlation with an ellipse resulting (e.g. from a quadratic failure criterion).

In Fig. 9 analogous results for a grain direction of $\varphi = 0°$ are presented. However, in addition to Fig. 8, the ratio of the principal stress components, $\sigma_2/\sigma_1$, and its propagation until fracture is illustrated. Considering this stress ratio, an almost proportional behavior was observed for tensile loading normal to the grain direction ($\sigma_2 > 0$). For compressive loading normal to the grain direction ($\sigma_2 < 0$), however, significant nonlinearities were detected. In order to eliminate stability problems as a reason of this phenomenon, experiments with a load history consisting of loading, unloading and reloading were carried out. Fig. 10 contains the stress-strain relationship for such a non-proportional experimentwith uniaxial compression normal to the grain direction ($\varphi = 0°$, $\kappa = 0{:}\,-1$). The results in this Figure are characterized by a deformation behavior which is well-known from the classical plasticity theory. Therefore, this theory can be used for the mathematical description of the stiffness degradation of wood under compressive loading.

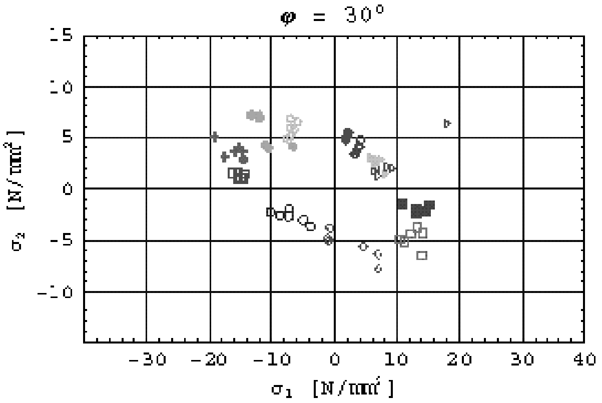


*Fig. 8 Biaxial strength envelope for grain direction $\varphi = 30°$*

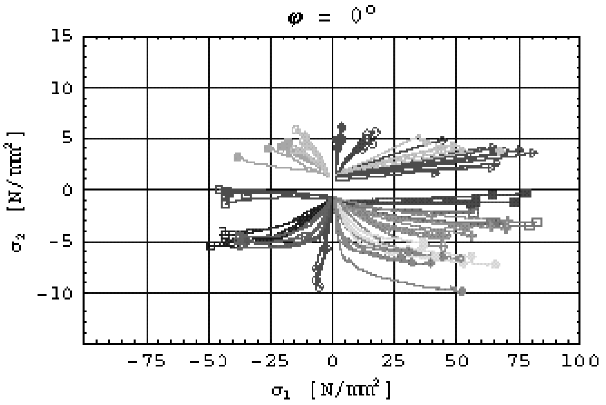


*Fig. 9 Biaxial strength envelope for grain direction $\varphi = 0°$*

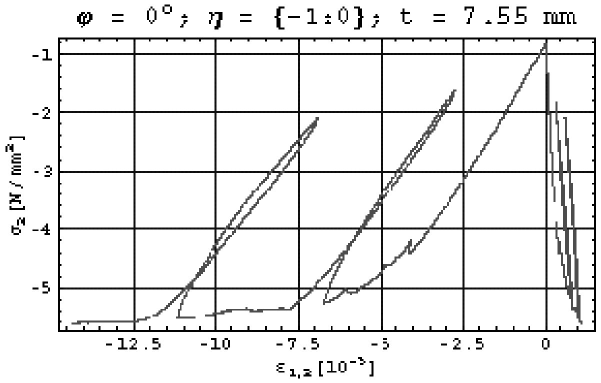


*Fig. 10 Stress-strain relationship for non-proportional loading*

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