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DYNAMIC PROPERTIES OF THE RAILWAY TRACK

The paper is focused on different aspects of track dynamics and train-track interaction. The dynamic modeling of railway track response and of the interaction of vehicle and track at low and mid frequencies, that are significant for the track deterioration, is presented. Dynamic properties of the track structure as a whole is described and evaluated including the dynamics of compound train-track system. The dynamic amplification resulting from the simulated passage of the locomotive of the type E 499.0 (85t) is presented.

1. Introduction

The use of a railway track structure by vehicles initiates dynamic loading, which gives vibrations of the interacting systems of the train and the track. The vibration behaviour of the railway track structure includes the wide frequency range, from the low frequencies (1–100 Hz) that corresponds the low-frequency vibration of track, through the mid frequencies (100–1500 Hz), and high frequencies (1500–5000 Hz) that corresponds transfer of the wheel-rail interaction forces and the sound radiation. These aspects deserve the attention and can be exploited as a performance indicator of the track structure status and a guiding instrument in track maintenance. One of the research goals at the Department of Structural Mechanics in the Track Mechanics field is to develop suitable methods for the evaluation of dynamic properties and the dynamic behaviour of a track structure that are focused on a conventional ballast track. With this purpose the program of theoretical and experimental works studying the interaction dynamic problems of a vehicle/track mechanical system focused on the track structure and the its dynamic behaviour has been undertaken and it comprises:

- Development of theoretical methods and mathematical models for solving interaction dynamic problems of the vehicle/track.
- Computer simulations and calculation to predict the dynamic response parameters of track – deformations, the strain and stress in the superstructure and in the substructure.
- In situ measurements of the response parameters of track structure.
- Static and dynamic laboratory tests of the track structure components – sleepers, fostering systems, resilient pads, the sleeper/ballast interaction, etc.

This paper is devoted to mathematical models and computer simulations of the dynamic behaviour of the track. Results of the theoretical dynamic analyses are compared with the experimental results.

2. Dynamic characteristics of the track structure

One way to investigate the dynamic properties of a railway track is to load the track with a sinusoidal force. At frequencies up to approximately 100 Hz this can be carried out by using a mechanical or hydraulic exciter. If one wants to investigate the track response at higher frequencies, the track may be excited by an impact load. The most employed excitation models for determination the track dynamic characteristic are shown in Fig. 1. The response of the track can be found in either the time or the frequency domain. The response of the track usually is searched as a displacement or acceleration at the point of excitation, the direct rail receptance, interaction forces, etc. The excitation models in Figs. 1 are appropriate for comparing the calculated and measured testing of the track – passages of characteristic vehicles, a vibrator or impulse hammer dynamic tests.

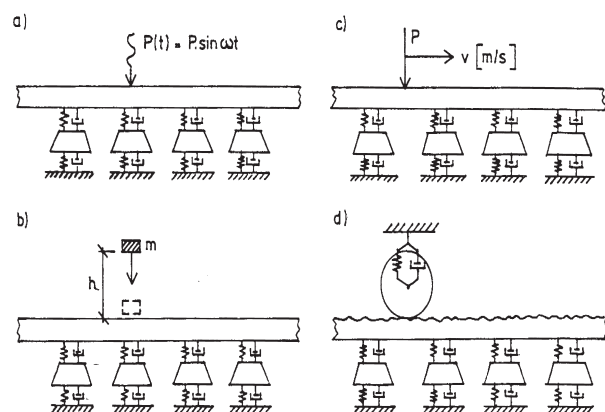


Fig. 1 Excitation models for the track structure

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In order to examine the effect of variable track stiffness conditions, the five characteristic vertical ballast track stiffness ${}^{(i)}k_b$, $i=1 \div 5$ were considered to model low, medium, and high track stiffness conditions. The mean values of the vertical ballast stiffness k_b and the vertical pad stiffness k_f are shown in Tab. 1.

Mean values of the modelled vertical stiffness of track Table 1

Track type (i)	Level of Support Stiffness	Vertical Ballast Stiffness ${}^{(i)}k_b$ [N/m]	Vertical Pad Stiffness k_f [N/m]	Damping coefficient C_f [Ns/m]
A	low	40.106	1	$150 \cdot 10^6$
			2	$60 \cdot 10^6$
B	Medium (1)	80.106	1	$150 \cdot 10^6$
			2	$60 \cdot 10^6$
C	Medium (2)	120.106	1	$150 \cdot 10^6$
			2	$60 \cdot 10^6$
D	high	220.106	1	$150 \cdot 10^6$
			2	$60 \cdot 10^6$
E	very high	480.106	1	$150 \cdot 10^6$
			2	$60 \cdot 10^6$

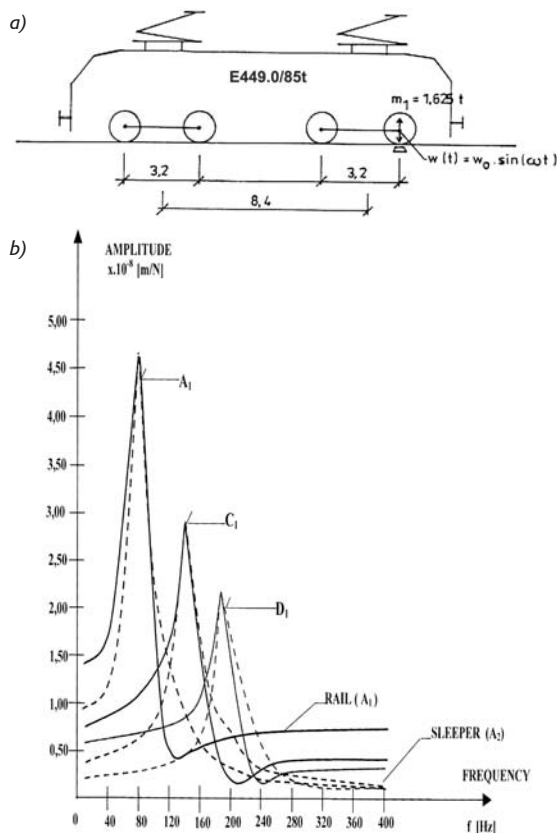


Fig. 2 The track response in the frequency range corresponding to the kinematics excitation - the vertical rail displacement amplitude $w_R(f)/P$ for the kinematics excitation by the first locomotive wheelset, for the track stiffness A_1 , C_1 and D_1

Some results for the stationary point kinematics track excitation by the locomotive wheelset (Fig. 2a) for the driving frequency $f(0-400$ Hz), applied on the first wheelset, and the track response - vertical rail displacement in the frequency range $w_R(f)$, are presented in Fig. 2b.

3. Modeling and simulation of the dynamic track in the time domain

The track behaviour under a moving vehicle is usually reproduced by an interactive dynamic model with three model components: the dynamic model of vehicle, the linear track model, and the stiffness of discrete rail supports.

- The conventional dynamic models represent a railway track as an Timoshenko beam, or if we neglected both the effect of shear and the effect of rotatory inertia we obtain the classical Bernoulli - Euler beam model rested on the Winkler linear elastic foundation governed by equation

$$EI \frac{\partial^4 w(x, t)}{\partial x^4} + \mu \frac{\partial^2 w(x, t)}{\partial t^2} + 2\mu\omega_b \frac{\partial w(x, t)}{\partial t} + \kappa w(x, t) = p(x, t) \quad (1)$$

where: $w(x, t)$ is the vertical displacement of the rail, EI is the bending stiffness of the rail, μ is the constant mass per unit length of the rail, ω_b is the circular frequency of damping of the rail, κ is the coefficient of Winkler foundation, and $p(x, t)$ is a external track load.

The force track loading is modeled as a vertical wheel force P_k , see Fig. 3. This model is deficient in several respects, in particular it neglects:

- The discrete nature of the support provided by the sleepers
- Elasticity of the railpad between rail and sleeper
- The ballast mass on the dynamic response
- The effect of variable track stiffness conditions.

This standard 1D Winkler beam on the elastic foundation modeling the track structure is schematic presented in Fig. 3.

- In the finite elements approach (FEA) the rail is modelled as an elastic beam on discrete supports. The 2D plain interaction model created in the Department of Structural Mechanics [6] is schematically shown in Fig. 5. This model is concerned with the study of low and mid frequencies $f = 0 - 300$ Hz. In this model the four beam elements belong to one sleeper bay. The track model which has been used (see Fig. 5) consists of the 36 sleepers, which are modeled as rigid bodies. The fastenings (pads) are modeled as a viscoelastic foundation with a linear (bilinear) stiffness k_f and viscous damping C_f . The ballast is also modeled as linear (bilinear) springs k_b and viscous dampers C_b in parallel. Resilient couples connecting the track components - the rail, sleepers, and the ballast are shown in Fig. 4.

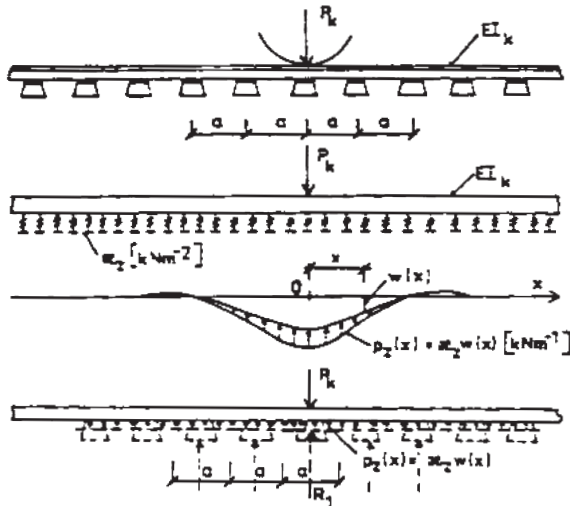


Fig. 3 Standard Winkler beam on the linear elastic foundation modelling the track structure

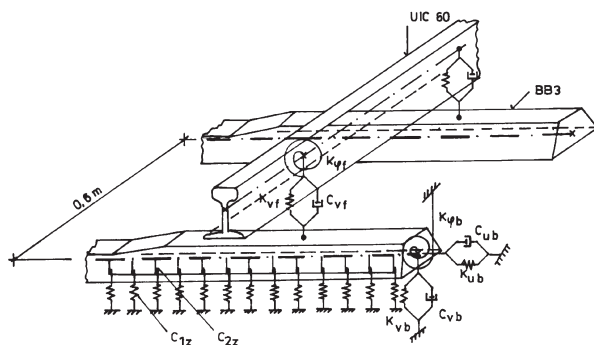


Fig. 4 The resilient couples connecting track components in the railway grid.

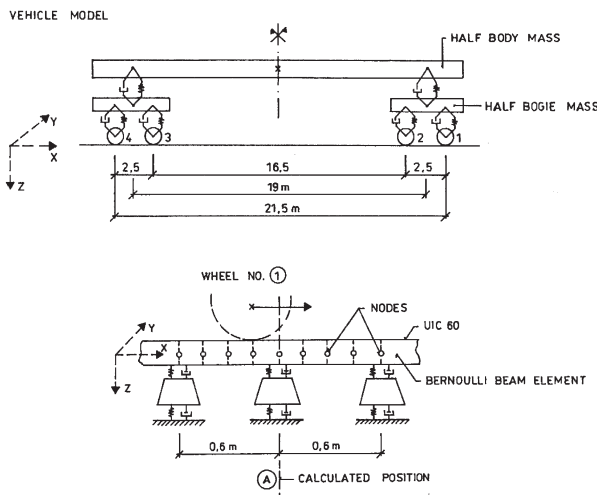


Fig. 5 The scheme of vehicle - track interaction model for FEA

The mechanical properties of the track components are modeled by a set of springs and dampers in one or two layers. The characteristics of springs and dampers can be determined by the laboratory load tests of these components or the field measurements in the typical track conditions.

The track behaviour under the dynamic loading conditions is reproduced by an interactive dynamic model with three model components:

- the dynamic model of vehicle,
- the linear track model,
- the stiffness of discrete rail supports.

The Finite Element Method is used for modeling of the track and the Composite Element Method is used for the modeling of vehicle. The resulting FEM motion equations of the track structure are:

$$[M]\{\ddot{U}\} + [C]\{\dot{U}\} + [K]\{U\} = \{P\} \quad (2)$$

where $[M]$, $[C]$ and $[K]$ are the mass, damping and stiffness matrices of the track structure respectively, and $\{P\}$ is the nodal load vector. $\{U\}$ is the nodal vertical displacement vector.

The vehicle is modelled by the Composite Element Method as a system described by three quantities: its mass $[m]$ or inherent properties, its internal force elements (springs and dampers $[c]$, $[k]$) and the generalised coordinates $\{u\}$. The equation of the vehicle mass parts then describes analogous process as in Eq.2:

$$[m]\{\ddot{u}\} + [c]\{\dot{u}\} + [k]\{u\} = \{F\} \quad (3)$$

where: $[m]$, $[c]$ and $[k]$ are the mass, damping and stiffness matrices of the vehicle system respectively, and $\{F\}$ is the nodal load vector. $\{u\}$ is the nodal vertical displacement vector.

Applying the above-mentioned equations of the finite element method and time-step integration, one can obtain simulation results including chosen amplitudes $Y(t)$ of the dynamic track response:

$$Y(t) = (w_R(t), w_S(t), R_{S-b}(t), M_R(t), \dots) \quad (4)$$

where: $w_R(t)$, $w_S(t)$ are the rail and sleeper deflections in the position above sleeper "A", see Fig. 5, $R_{S-b}(t)$ is the sleeper-ballast vertical interaction force under sleeper "A", and $M_R(t)$ is the rail bending moment, etc.

The approach can be helpful in clarifying the influence of vehicle speed, stiffness of subgrade, stiffness and damping of fastenings on the dynamic response of the track. The example of simulation output in the time domain applied for the locomotive Skoda E 499.0 (85t) operating by speeds of 20 m/s are presented in Fig. 6.

- In applying the commercial FEA programs the track system can be suitable modeled as a plain grid with resilient couplings between the rail, sleepers, and the subgrade. The five elastic constant $[k]$

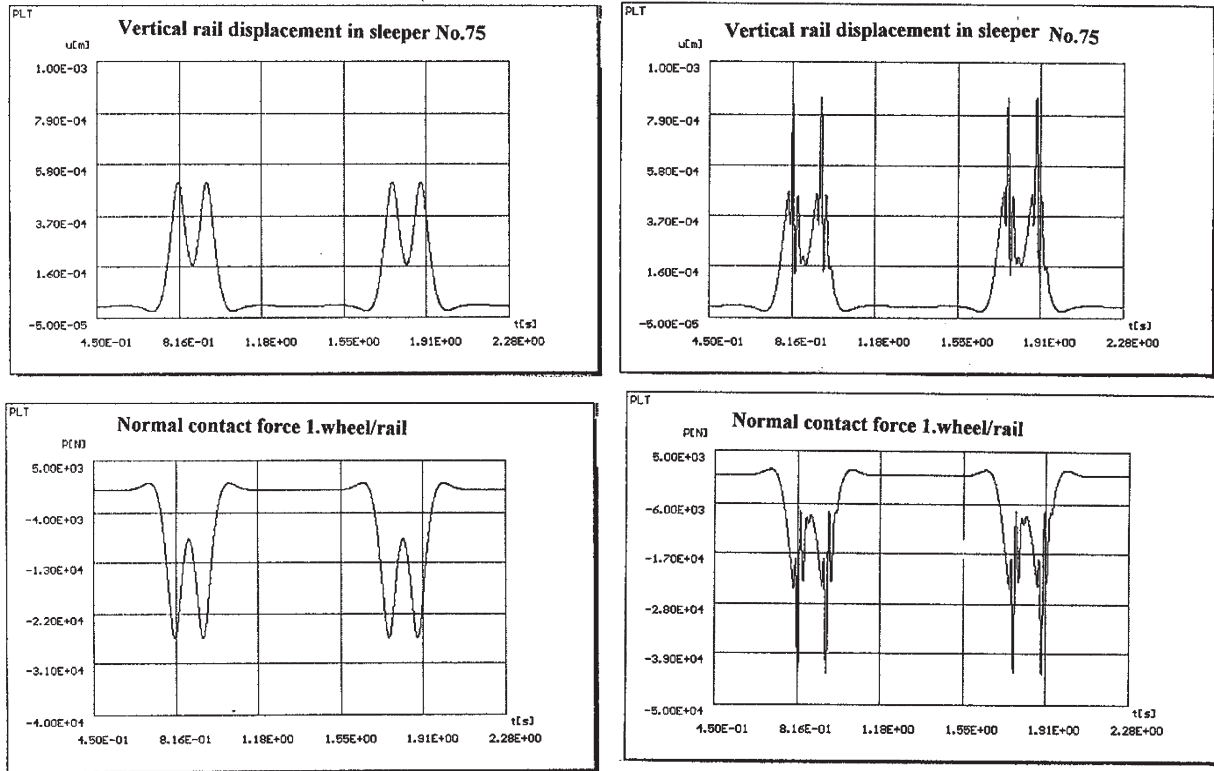


Fig. 6 Vertical dynamic sleeper displacements and the normal contact force for the stationary point force excitation by the locomotive wheelset:
 a) the locomotive smooth passage, $v = 20$ m/h, b) The locomotive passage across the single cos. shape irregularity, $v = 20$ m/h

characterising the resilient properties of subgrade entering as an input data in the computer system FEAT 2000, [7] - three constants k_{1x}, k_{2x}, k_{3x} described the vertical stiffness of subgrade and two constants k_{1y}, k_{2y} described the shear stiffness of subgrade. In regard to the variability of vertical stiffness of the subgrade during the track exploiting, the regular estimation of these constant has the fundamental significance for the respectable results. The model of the railway grid with the resilient couplings modeled in the computer system FEAT 2000 is shown in Fig. 7.

3.1 Dynamic coefficient

A moving vehicle on a track with a stochastic rigidity of the substructure in the vertical direction $\{^{(i)}k_j\}$ generates deflections and stresses in the track structure that are generally greater than those caused by the same vehicle load applied statically or moving on the track with the constant rigidity of the substructure. This actual dynamic wheel loads are approximately represented by equivalent quasi-static load. The simple approach for determining the wheel load $P_{w,dyn}$ uses empirical equations containing an impact factor δ_{dyn} and a wheel load $P_{w,st}$ can be expressed in the form

$$P_{w,dyn} = \delta_{dyn} \cdot P_{w,st} \quad (5)$$

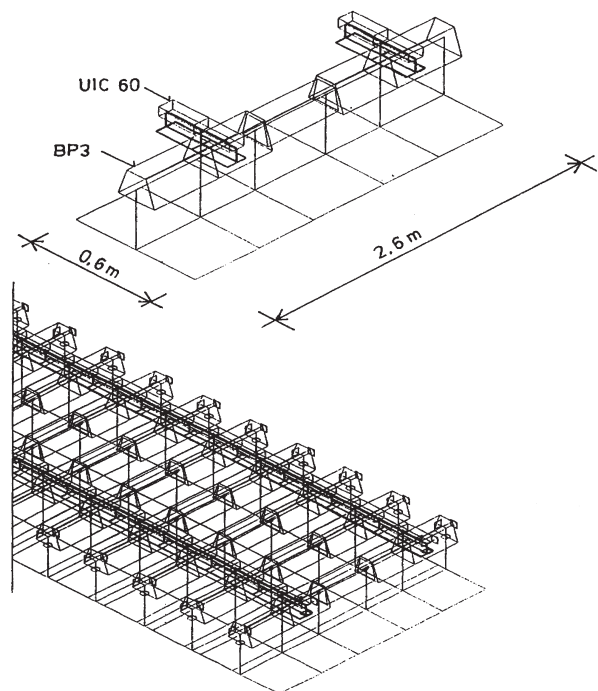


Fig. 7 The railway grid model including resilient component couplings, modeled in the computer program FEAT 2000

The dynamic amplification δ_{dyn} generally has a stochastic character and can be defined as a ratio of the maximum dynamic response of a quantity Y_{dyn} , for example $Y_{dyn} = w_{w,dyn}$ - the rail dynamic deflection, $w_{w,dyn}$, etc. to the static deflection of a quantity Y_{st} .

3.2 Results of numerical simulation

The passage of each vehicle wheelset induces amplitudes of the observed quantity $^{(i)}Y_{dyn(road)}$ and these results may be treated statistically. Thus, the histograms of these amplitudes may be exploited for the statistic expected values of the dynamic coefficient δ_{dyn} for the rail deflection, bending moments, etc.

$$\delta_{dyn} = \frac{^{(i)}Y_{dyn(road)}}{Y_{st}} \quad (6)$$

The example evaluating of the dynamic coefficients δ_{dyn} from chosen quantities w_{dyn} and P_{S-b} (for the track B1 considering the constant stiffness of supports) are presented in Tab. 2.

Dynamic coefficient δ for the constant stiffness of support, $k_{b,j} = const.$ Table 2

Track type	Amplitude response			Dynamic coefficient	
	Y_{st}	Y_{dyn}		$\delta_{dyn} = \frac{Y_{dyn}}{Y_{st}}$	
		20 m/s	50 m/s	20 m/s	50 m/s
Rail deflection w_{dyn} [mm]	0.796	0.821	0.908	1.03	1.14
Sleeper-ballast force P_{S-b} [kN]	41.5	42.9	47.7	1.03	1.15

Using the mean values $\mu_{\{Y\}} = \bar{Y}$ of the response quantities \bar{Y} and the standard deviation $\sigma_{\{Y\}}$, the dynamic coefficient δ_{dyn} may be evaluated. The dynamic coefficient δ_{dyn} for the constant stiffness of supports, see Tab. 2, is small for the low vehicle speed and it increases with vehicle speed.

The effect of a stochastic rail support stiffness can not be ignored, particularly for the higher vehicle speed, see [4, 5] and some results are in Tab. 3. The results obtained confirm that the higher values standard deviations of the support stiffness can be one of the important factors causing an intensive dynamic response of track components.

4. Conclusion

The purpose of simulating the dynamic track behaviour was to assess the dynamic behaviour of rail $w_R(t)$, sleepers $w_S(t)$ and the interaction force the rail - the sleeper $R_{S-b}(t)$ under the train passages, especially the locomotive passage of the type E 499/85 t in operational conditions. In this paper are presented the simulation approaches for the typical locomotive passage only.

The response of a railway track that includes the uncertainties in the vertical track supports stiffness subjected to moving railway vehicle is solved by the finite element method and time-step integration. Monte Carlo simulation was applied for these cases to estimate the dynamic track response to variations of the subgrade stiffness that was simulated as a stationary randomly distributed ballast stiffness with a standard uniform distribution and a normal distribution. The herein presented simulation results of the dynamic interaction are concerned in the low and mid frequencies, say 0 - 300 Hz, and they are applied to the track with different vertical stiffness. The dynamic amplification resulting from the simulated passage of a vehicle over the simulated track section can be calculated as the ratio of the maximum dynamic response Y_{dyn} (deflection w , bending moment M , etc.) to the static response Y_{st} on given track stiffness level. Some obtained results can be summarised as follows:

- The dynamic response results due to the tracks with the constant stiffness of supports (deterministic cases) show a low dynamic amplification $\delta = 1.05 - 1.2$, for the track response quantities $Y = (w, P, M, \text{etc.})$ in relation to the vehicle speed c .
- The dynamic response due to the stationary randomly distributed stiffness of supports $k_{b,j}$, with the standard uniform and normal distribution, follows the results of the static analysis and they show how the vehicle speed influences the track response.

Dynamic coefficient δ for the Uniform distribution of support stiffness $k_{b,j}$,

Table 3

Track type	Support stiffness $k_{b,j} \in (60.10^6 - 100.10^6 \text{ N/m})$					
	$\delta_{dyn} = \frac{Y_{dyn}}{Y_{st}}$		$\delta_{dyn} = \frac{(\bar{Y}_{dyn} + \sigma_Y)}{Y_{st}}$		$\delta_{dyn} = \frac{(\bar{Y}_{dyn} + 2\sigma_Y)}{Y_{st}}$	
	20 m/s	50 m/s	20 m/s	50 m/s	20 m/s	50 m/s
Rail Deflection w_{dyn} [mm]	1.03	1.15	1.18	1.33	1.33	1.51
Sleeper-ballast Force P_{S-b} [kN]	1.01	1.14	1.17	1.5	1.32	1.50

In these cases the effect of the stochastic rail support stiffness can not be ignored, in particular for the higher vehicle speeds.

- While the mean value of the response amplitudes \bar{Y} and the corresponding dynamic coefficients δ_{dyn} attain no high values for

these cases, the individual response amplitudes $\{Y\}$ can attain values that can not be ignored. The standard deviations of support stiffness σ_γ is an important factor affecting the dynamic response on a given level of support stiffness.

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