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NOTE ON THE COORDINATION OF PERIODIC PROCESSES IN TRANSPORTATION SYSTEMS

The article deals with a coordination of periodic processes in transport systems. It introduces some basic mathematical models useful in process optimization. It also shows some examples of everyday transport practices where this optimization can be used.

1. Introduction

Users of transport systems often declare their interest in velocity. However, it does not mean technical velocity of transport means. They concern the "velocity of displacement", i.e. the distance between the origin and destination divided by the transport time. And it is often true that this value is strongly influenced by coordination between transportation processes.

Let us consider several examples:

A. A passenger travels from the village V to the town W using first a local train from V to a station S and afterwards an express train from S to W. It is obvious that the travel time from V to W is strongly influenced by the waiting time for the train changing in S and, consequently, by the coordination of the transport processes of local and express train operation.

B. A passenger walks from his house H to an urban bus stop S_1, then he/she travels by bus from the stop S_1 to the stop S_2, using any from the two routes r_1, r_2 operating between S_1 and S_2. It is obvious that the total duration of the trip from H to S_2 is strongly influenced by the waiting time for a bus at the stop S_1 and, consequently, by the coordination of the transport processes on the two routes.

C. A wagon is loaded at a station A and its destination is the station B. However, it has first to use a train to a marshalling yard Y and after some manipulations there to use another train from Y to B. It is obvious that the total duration of the trip from A to B is strongly influenced by the manipulation time in Y and, consequently, by the coordination of the processes in Y.

D. A person drives his car from his home H to his office O passing through a signalized road intersection I. It is obvious that the total duration of the trip from H to O is strongly influenced by the waiting time in front of the junction I and, consequently, by the coordination of the processes at I.

If we consider the cases A., B. and D. we can imagine that the displacement activities may be more complicated:

A. There may exist many local trains coordinated with several express trains in several stations.

B. There may exist many routes coordinated at many common legs.

D. There may exist many processes at several junctions to be coordinated.

The complex problems of optimal coordination can be a bit simplified introducing a periodicity:

A. To introduce a periodic timetable (a "Takt Fahrplan" in German), e.g. to repeat the departures after each hour.

B. To introduce a periodic time table, e.g. to repeat the departures after each 12 minutes.

C. To introduce a periodic schedule, e.g. to repeat the processes each day (it was already done in the major part of marshalling yards).

D. To introduce a "cyclic" mode of operation, i.e. to repeat the green signals e.g. each 80 seconds (a big part of junctions work in a periodic regime now).

In the sequel we shall try to formulate the coordination problems in a general mathematical way.

2. Basic mathematical models

One can find many different periodic point processes (more precisely: time-point processes) in transportation systems, e.g.:

A. The departures of express trains from the station S_1 for the station S_2 are scheduled for 6:42, 7:42, 8:42 etc. This defines a point process p_2 = 6:42, 7:42, 8:42, ... = 6:42 + k × 1:00, k = 1, 2, ... (the reason for using p2 will be seen later)

B. The departures of urban transport buses from the stop S_1 for the stop S_2 are scheduled for 6:06, 6:18, 6:30, 6:42, etc. p_1 = = 6:06, 6:18, 6:30, 6:42, ... =. 6:06 + k × 0:12, k = 1, 2, ...

C. Wagon collection process on the sorting siding s_1 of a marshalling yard is concluded each 6 hours at 3:20, 9:20, 15:20, 21:20. p_1 = 3:20, 9:20, ... = 3:20 + k × 6:00, k = 1, 2, ...

D. At a signalized road intersection I, the green light start each 80 seconds for the given stream S_1 of vehicles e.g. p_1 = = 5:00:12 + k × 0:1:20, k = 1, 2, ...

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Usually, the periodic point processes are not isolated. On the contrary, there exist sets of mutually influenced processes and, usually again, the processes have either the same period, or the least common multiple of the periods, which is not very much greater than they are. For instance:

A. Besides the process p2 we have another process $p_1 = 6{:}28$, $6{:}58$, $7{:}28$, ... $= 6{:}28 + k \times 0{:}30$ representing the arrivals of local trains to the station S_1 from the station S_3. In this case, the corresponding periods are $p_2 = 1{:}00 = 60$ min., $p_1 = 0{:}30 = 30$ min., the common multiple $p = 60$ min. $= 2 \times p_1 = 1 \times p_2$.
B. Besides the process p_1, corresponding to a route r_1 we have another process $p_2 = 6{:}00, 6{:}20, 6{:}40, ... = 6{:}00 + k \times 0{:}20$ representing the departures of buses of another route r2 from the stop S_1 for the stop S_3 via the stop S_2. In this case, the corresponding periods are $p_1 = 0{:}12 = 12$ min., $p_2 = 0{:}20 = 20$ min., the common multiple $p = 60$ min. $= 5 \times p1 = 3 \times p2$.
C. Besides the process p_1, corresponding to the sorting siding s_1 we have another process $p_2 = 1{:}30, 9{:}30, ... = 1{:}30 + k \times 8{:}00$ corresponding to the sorting siding s_2. In this case, the corresponding periods are $p_1 = 6{:}00 = 6$ *h*., $p_2 = 8{:}00 = 8$ *h*., the common multiple $p = 24$ min. $= 4 \times p_1 = 3 \times p_2$.
D. Besides the process p_1, corresponding to the stream S_1 we have another process $p_2 = 5{:}00{:}45 + k \times 0{:}1{:}20$, $k = 1, 2, ...$ corresponding to another stream S_2, which is in collision with S_1. Here $p_1 = p_2 = p = 80$ seconds.

2. Coordination

Having two processes at the same place it is quite natural to require some type of coordination between them. In general, we can meet different requirements:

2.1. Single and simple linking. We speak about *single linking* if we have two processes p_1, p_2 with only one linking between them, e.g. p_2 linked to p_1, symbolically $p_1 \rightarrow p_2$. We call this linking simple if both processes have the same number n of time-points in one period p and the process $p_2 = t_{21}, t_{22}, ..., t_{2n}$ $(+\ kp)$ is linked to the process $p_1 = t_{11}, t_{12}, ..., t_{1n}$ $(+\ kp)$. The quality of this linking can be expressed by the differences $d_1 = t_{21} - t_{11}$, $d_2 = t_{22} - t_{12}$, ..., $d_n = t_{2n} - t_{1n}$ put into an objective function $f(d_1, d_2, ..., d_n)$, e.g.

$f_1(d_1, d_2, ..., d_n) = \min\{d_1, d_2, ..., d_n\}$ (greater is better)
$f_2(d_1, d_2, ..., d_n) = \max\{d_1, d_2, ..., d_n\}$ (smaller is better)
$f_3(d_1, d_2, ..., d_n) = \max\{d_1, d_2, ..., d_n\} - \min\{d_1, d_2, ..., d_n\}$ (smaller is better)
$f_4(d_1, d_2, ..., d_n) = d_1^2 + d_2^2 + ... + d_n^2$ (smaller is better).

Among the abovementioned examples only the A can serve as the illustration of single linking, but, unfortunately, it is not simple (we have 2 arrivals within 60 min. but only one departure).

2.2. Double and multiple linking. We speak about *double linking* if:
a) We have both $p_1 \rightarrow p_2$ and $p_2 \rightarrow p_1$.
b) There exists another pair $p_3 \rightarrow p_4$.

In the case a) we speak about *double mutual linking* of the pair p_1, p_2.

We speak about *multiple linking* if there exist more than one linked pair (i.e. double linking is a particular case of multiple linking). We suppose we are given a set of time-point periodic processes $P = \{p_1, p_2, ..., p_m\}$ with periods $p_1, p_2, ..., p_m$ and the common period $p = \mu(p_1, p_2, ..., p_m)$ = the least common multiple of the periods $p_1, p_2, ..., p_m$. The mutual linking could be expressed by means of a *linking digraph* (= oriented graph) $G = (V, A)$, where the vertex set $V = \{1, 2, ..., m\}$ represents the processes $p_1, p_2, ..., p_m$, the arc set A represents linking, i.e. the *arc* $a = (i, j) \in A$ represents linking p_j to p_i i.e. $p_i \rightarrow p_j$.

Writing the time-point one has to take into account the common period p, i.e. if a time-point is represented by a number t_{ik} then $t_{ik} \in \{0, 1, ..., \mathrm{p}\}$ must hold; if not, then it must be reduced mod p.

2.3. General single linking. We speak about general single linking $p_1 \rightarrow p_2$ if it is not simple. In the case of general single linking the processes p_1, p_2 may have different numbers n_1, n_2 of time-points in one period p and the differences d_i may be calculated from some selected n-tuples $t_{11}, t_{12}, ..., t_{1n}, t_{21}, t_{22}, ..., t_{2n}$, $n \leq n_1$, $n \leq n_2$.

Especially, if the linking $p_1 \rightarrow p_2$ intends to express a *train changing*, we denote T_1 the set of all time-points (= arrivals to the change station plus some time for walking from one train to another) of the process p_1 in one period p and T_2 is the set of all time-points (departures from the change station) p_2, but moreover both the sets T_1, T_2 are extended by adding the first time-point from the next period (i.e. we add the time-point $t + p$ where t is the first time-point from the period p). Then the number n is the maximum natural number allowing the n-tuples $t_{11}, t_{12}, ..., t_{1n}, t_{21}, t_{22}, ..., t_{2n}$ to have the following properties of "closeness": $t_{1i} = \max\{t \in T_1 : t \leq t_{2i}\}$, $t_{2i} = \min\{t \in T_2 : t \geq t_{1i}\}$. If in such a manner $t_{1n} = t_{11} + p$ and $t_{2n} = t_{21} + p$ then we omit t_{1n}, t_{2n} and we put $n - 1$ instead of n.

In our example A we have $T_1 = \{28, 58, 88\}$, $T_2 = \{42, 102\}$ corresponding to the local train arrivals 6:28, 6:58, 7:28 and the express train departures 6:42, 7:42. Obviously, the first value of $n = 2$, $t_{11}, t_{12} = 28, 88$, $t_{21}, t_{22} = 42, 102$. However, $p = 60$ and both $88 = 28 + p$, $102 = 42 + p$. Thus, we omit 88, 102 and put $n = 1$. $t_{11} = 28$, $t_{22} = 42$, $d_1 = 14$.

2.4. General multiple linking. We suppose we are given a set of time-point periodic processes $P = \{p_1, p_2, ..., p_m\}$ with periods $p_1, p_2, ..., p_m$ and the common period $p = \mu(p_1, p_2, ..., p_m)$ and the mutual linking expressed by means of a linking digraph $G = (V, A)$. In the case of general multiple linking we suppose there exists a general rule determining the number n, the values $d_1, d_2, ..., d_n$ and the objective function f expressing the quality of linking by means of the value $f(d_1, d_2, ..., d_n)$.

In practice it can happen that these processes are originated in different locations and their influence represented by a relation $p_i \rightarrow p_j$ "works" in a third location which needs some equalization of time-points. Usually, we use a value $o(i, j)$ said the *offset* which has to be added to the time-points of the process p_i before their comparison with the time-points of p_j.

The practical meaning of the offset can be demonstrated on our examples:

A. Let us suppose that the departures of local trains from the station S_3 are 5:38, 6:08, 6:58, ... = 5:38 + $k \times$ 0:30 and let the running time from S_3 to S_1 be $r(S_3, S_1) = 50$ min. Then the arrivals to S_1 are 6:28, 6:58, 7:28, ... = 6:28 + $k \times$ 0:30 as we supposed above. Moreover, let the departures of express trains from their origin station S_0 be 5:32, 6:32, 7:32 etc., i.e. 5:32 + $k \times$ 1:00, just $r'(S_0, S_1)$ = 1:10 h. = 70 min. before their departures from S_1. Then we can consider the original processes p_1 = 5:38 + $k \times$ 0:30, p_2 = 5:32 + $k \times$ 1:00, the offset $o(1, 2) = r(S_3, S_1) - r'(S_0, S_1) + w_{12} = 50 - 70 + 5 = -15$ min. Hence the process p_2 = 5:32 + $k \times$ 1:00 = $= 32 + 60k$ (with one departure during the common period $p = 60$ min.) will be linked to the reduced process p_{r1} = 5:38 + + $k \times$ 0:30 + $o(1, 2) = 38 + 30k - 15 = 23 + 30k$ (with two departures during the common period 60 min.). Further calculations will be similar to the ones at the end of the part 2.3.

B. Similarly, the original processes can be represented by the departures from the terminals of the routes, but the offset will be exactly the difference of running times from the terminals to S_1 without any further correction. Let, for the simplicity, S_1 be the terminal for both routes. The "general rule" is the following: Let $T_1 = \{t_1, t_2, ..., t_n\}$ be the set of all departures of all routes from S_1 to S_2 within one period p. Suppose that $t_1 \leq ..., \leq t_n$. Let $t_{n+1} = t_1 + p$. Then $d_i = t_{i+1} - t_i$, $i = 1, ..., n$. The values $d_1, d_2, ..., d_n$ represent the waiting intervals of the passengers using the segment S_1, S_2 only.
In our example $d_1, ..., d_8$ = 6, 12, 2, 10, 10, 2, 12, 6 and:
- $f_1(6, 12, 2, 10, 10, 2, 12, 6) = \min\{6, 12, 2, 10, 10, 2, 12, 6\} = 2$ expresses the danger of collision of two subsequent vehicles at the same stop,
- $f_2(6, 12, 2, 10, 10, 2, 12, 6) = \max\{6, 12, 2, 10, 10, 2, 12, 6\} = 12$ expresses the maximum waiting time of a passenger,
- $0{,}5qf_4(6, 12, 2, 10, 10, 2, 12, 6) = 0{,}5q(36 + 144 + 4 + 100 + 100 + 4 + 144 + 36) = 284q$ expresses for one period the total waiting time (in minutes) of the q passengers, boarding the vehicles during one minute.

C. On the contrary, here no running time will have to be considered. Instead, the offset $o(i, j)$ will represent the transfer time for employees and engines to move from the i-th siding to the j-th one after having finished the works transforming wagons into a train. The general rule for the calculation of the values di is similar to the previous one. In our example, neglecting offsets, we have (in minutes) $d_1, ..., d_7$ = 110, 360, 10, 350, 130, 230, 250, but the only objective function having a practical sense is:
- $f_1(d_1, ..., d_7) = \min\{110, 360, 10, 350, 130, 230, 250\} = 10$
It expresses the shortage of time necessary for the train creation from the wagons on the siding s_1 before the start of the same works at the siding s_2.

D. If the processes p_i and p_j work at the same intersection then the offset $o(i, j)$ will represent the duration of green signal for the stream Si plus the clearing time. If they work at different intersections then the running time between them will have to be added.

3. Coordination for changes

Let us turn to our examples. B) concerns the well known problem of coordination of public transport on common legs. C) deals with freight train formations and the methods of solution are similar to the previous ones. D) is the well known problem of signalised intersections. All three are described by many authors in many books and papers, see. e.g. the monograph [1]. One can say that the available methods satisfy the practical needs.

On the other hand the problem A) cannot be considered satisfactorily solved. In [1] and [2] one can find a heuristics and a linear programming model for time shift optimization, having given the set of trains operating on a general network. Other authors, e.g. [3], study the same problem, but limited to the "herring-bone type of network" using congruency calculations.

However, the authors have not yet met any paper dealing with the coordination problems, *optimizing*:

a) *the size of trains* for the given number of trains during the common period together with their *time shifts*
b) *the number of trains* during the common period together with their *size and time positions*.

The authors hope they have an idea of solution of the problem a) using mathematical programming. The problem b) seems to be open.

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