

## CONTACT AREA AND NORMAL STRESS DETERMINATION ON RAILWAY WHEEL / RAIL CONTACT

*It is important to study the field of railway wheel /rail contact area from the point of view of stress distribution for further investigation of running behavior of a vehicle when running on a rail track. There is a contact of two spatial geometrically defined bodies, which have characteristic material properties, by a power effect of a vehicle through the wheel on the rail. The contact point is transformed into the contact area as a result of the pressure of the bodies against each other. It is possible to acquire the image of the contact area and the stress which appears at the contact as a result of the application of a known calculation method or to set up own calculation methods. The paper deals with theoretical grounds of two computational methods. They are the Hertz method and strip method (according to Johnson [5]). The results of the contact area and normal stress numerical computation made by authors of the paper are stated and compared with the above mentioned methods and the results of the contact area and normal stress numerical computation made by the authors of the paper are stated and compared with the methods in references [7].) The contribution of the paper is to present the authors' computational results and compare the results with published results of other authors [7]. The authors reached the results with the support of their own computational program. The results are the base for other tangential contact stress research computations and base for other vehicle dynamics simulation computation in the future.*

### 1. Introduction

An important parameter influencing the power effect of a wheel on the rail is the size and shape of the contact area as well as the normal stress distribution which has the impact on it. Nowadays various methods are used to find out the size of contact areas and stresses. It is necessary to mention the Hertz method as one of the oldest and up to date used methods. It provides acceptable results for a large area in spite of many simplifications. Another computational procedure is the stripe method which is, thanks to its results, close to reality and it is used in the following calculations. Nowadays, another group of calculation program systems (ANSYS, MSC.MARC, ADINA...) is used in certain situations. The systems work on the base of finite elements method theory. Searching of the solution of contact problems with the help of finite element method is not the subject of the paper.

### 2. Hertz method

Hertz method [4, 5, 6, 9] belongs to widely used methods of contact area and contact stress determination even in the present. We can simplify the wheel and rail contact as a contact of two cylindrical areas. The rail head surface, with the diameter  $R_x^{(2)} = \infty$ ,  $R_y^{(2)}$ , presents one of the cylindrical areas. The running tread of the wheel, with the diameter  $R_x^{(1)}$ ,  $R_y^{(1)}$ , builds a second idealized cylindrical area (conical area in reality). These two skew cylinders with mutually perpendicular axes touch each other at the contact area. This happens by the effect of the vertical wheel force  $Q$ . The Hertz solution of the two bodies contact problem comes from a presupposition that:

material is homogenous, isotropic, material behaves elastically, displacements and stresses are bordered in space, the contact area is small with regard to the surface, the area is positive and plain, there is no surface slip, and there is no spin. [7]. In spite of the fact that many presuppositions are not fulfilled, or they are fulfilled only partially, the theoretical calculations show sufficient agreement with experiments.

$$A = \frac{1}{2} \cdot \left( \frac{1}{R_x^{(1)}} + \frac{1}{R_x^{(2)}} \right) \quad B = \frac{1}{2} \cdot \left( \frac{1}{R_y^{(1)}} + \frac{1}{R_y^{(2)}} \right) \quad (1)$$

At first principal curvatures are calculated from the known profiles curvatures diameters.

The size of the half axes  $a$ ,  $b$ , of the contact area is given by the following formulas with the usage of the Hertz method. It is used in practice [6]:

$$a = \alpha \cdot \sqrt[3]{\frac{3 \cdot Q \cdot (1 - \nu^2)}{2 \cdot E \cdot (A + B)}} \quad b = \beta \cdot \sqrt[3]{\frac{3 \cdot Q \cdot (1 - \nu^2)}{2 \cdot E \cdot (A + B)}} \quad (2)$$

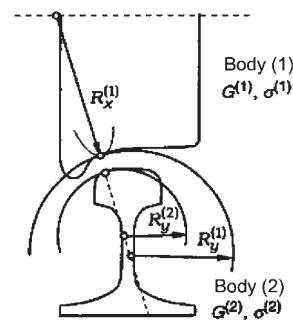


Fig.1. Curvature diameters

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Where  
 $Q$  - is vertical wheel force,  
 $E$  - is modulus of stiffness,  
 $\nu$  - is Poisson's ratio,  
 $A, B$  - principal diameters of the wheel and rail curvature.

Body radius in contact point  
 $R_x^{(1)}$  - wheel (x-z cross section area)  
 $R_y^{(1)}$  - wheel (y-z cross section area)  
 $R_x^{(2)}$  - rail (x-z cross section area)  
 $R_y^{(2)}$  - rail (y-z cross section area)

Constants  $\alpha$  and  $\beta$  in the formulae are given in tables and depend on the angle  $\delta$ , which is defined:

$$\delta = \arccos \frac{|B - A|}{A + B}$$

From which arises:  $\cos \delta = \frac{|B - A|}{A + B}$  (3)

Values  $\delta, \alpha$  and  $\beta$  are given in the table of appendix [6].

The normal stress  $p$ , which is distributed in the form of ellipsoid, appears by the vertical wheel force  $Q$  effect, according to the formula:

$$p = p_0 \cdot \sqrt{1 - \left(\frac{x}{a}\right)^2 - \left(\frac{y}{b}\right)^2}$$

where is  $p_0 = p_{max} = \frac{3}{2} \cdot \frac{Q}{\pi \cdot a \cdot b} = 1.5 p_{str}$ . (4)

There is an example of two cylinder contact in Fig. 2 and Fig. 3, for more simplicity. One of the cylinders presents a wheel and has a diameter  $R = 460$  mm, the second cylinder presents the rail and has the curvature diameter of 300 mm. The axes of both cylinders are perpendicular. If they were not perpendicular, we would have to take into consideration their angle when calculating relative curvatures  $A$  and  $B$ .

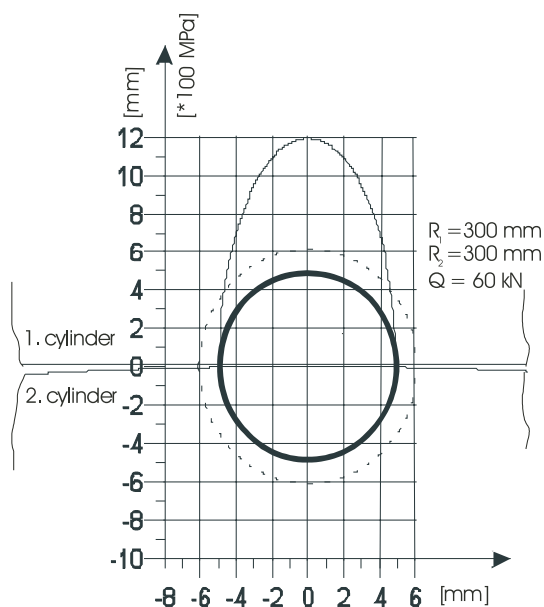


Fig. 2. Contact area shape and normal stress of cylinders with diameters 300 mm

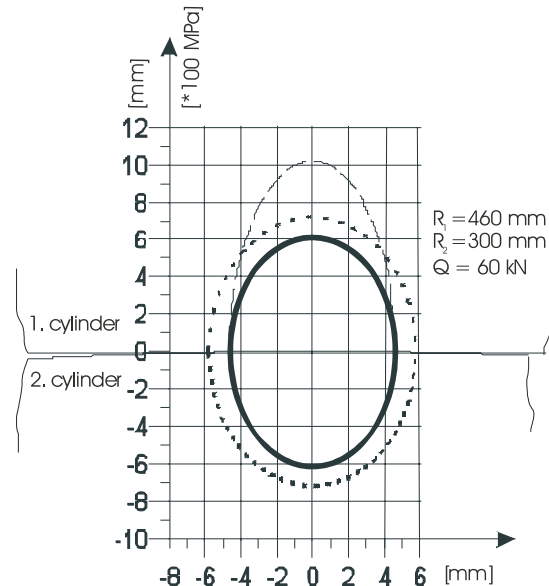


Fig. 3. Contact area shape and normal stress of two cylinders with diameters of 460 and 300 mm

The situation is changed when the real wheel and rail profiles are taken into account. The contact area acquires a non-elliptical shape with regard to a rapid change of the rail profile curvature diameter.

### 3. Strip method

Strip method [1, 4, 5, 6, 7, 9] presupposes quasi-static rolling. The principal idea of the theory is to take into consideration slim contact areas in the y-direction.

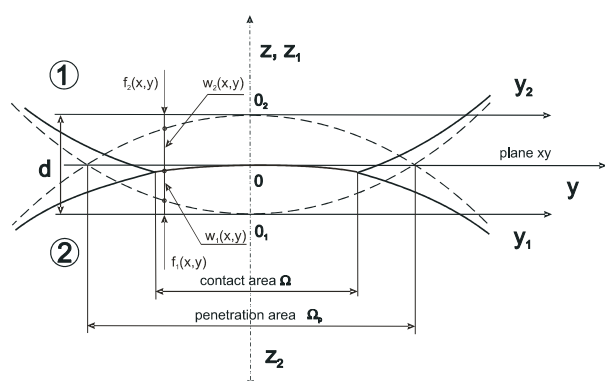


Fig. 4. Coordinate body system at the contact

In Fig. 4 there are two bodies in contact. Geometrical parameters (of railway wheel and rail) should be very similar in reality, deformation zones are similar too. In spite of this fact the parameters (the displacements  $w_1$  and  $w_2$ ) in Fig. 4 are rather different

for better understanding of theory. In fact the contact area should be plane (parallel with the x-y plane).

The method presupposes the existence of two rotating bodies 1 and 2 with surfaces  $S_1$  and  $S_2$ . The bodies touch in the point 0, which is at the same time the beginning of their spatial coordinate systems. The axes  $x$  and  $y$  determinate the horizontal base. We will mark the horizontal coordinate as the  $z$  - axis. If there is no influence of a normal force  $Q$ , then exclusively geometrical binding between the bodies exists.

If the bodies are pressed against each other by the normal force  $Q$ , there is a deformation and a contact area  $\Omega$  between the bodies appears instead of a contact point.

- The geometrical profile shape of the first body surface will be marked  $f_1(x, y)$ , the geometrical profile shape of the second body surface will be marked  $f_2(x, y)$ .
- The elastic displacement in the  $z$ -axis direction caused by the deformation of the first body surface will be marked  $w_1(x, y)$ , the displacement in the  $z$ -axis direction caused by the deformation of the second body surface will be marked  $w_2(x, y)$ .
- The displacement of bodies centers against each other in the axis- $z$  direction will be marked  $d(x, y)$ .
- The perpendicular distance between the points of the deformed bodies surfaces will be marked  $\delta(x, y)$ .

$$\delta(x, y) = f_1(x, y) + w_1(x, y) + f_2(x, y) + w_2(x, y) - d(x, y) \quad (5)$$

A function  $\delta(x, y)$  determines the dependence of the deformed bodies surfaces position. It has the zero value in the contact point.

$$\begin{aligned} \delta(x, y) &= 0 && \text{in the case of the } (x, y) \in \Omega \\ \delta(x, y) &> 0 && \text{in the case of the } (x, y) \notin \Omega \end{aligned} \quad (6)$$

A normal stress effects only in the sphere of the contact area  $\Omega$ . Boundary conditions for the influence of the normal stress:

$$\begin{aligned} \sigma_z(x, y, 0) &= -p(x, y) && \text{in the case of the } (x, y) \in \Omega \\ \sigma_z(x, y, 0) &= 0 && \text{in the case of the } (x, y) \notin \Omega \end{aligned} \quad (7)$$

Body surface displacements from the elastic deformation  $w_1(x, y)$  and  $w_2(x, y)$  are calculated from the normal stress distribution in the contact [9].

$$\begin{aligned} w_1(x, y) &= \frac{1 - \mu_1^2}{\pi \cdot E_1} \cdot \iint_{\Omega} \frac{p(x', y')}{\sqrt{(x - x')^2 + (y - y')^2}} dx' dy' \\ w_2(x, y) &= \frac{1 - \mu_2^2}{\pi \cdot E_2} \cdot \iint_{\Omega} \frac{p(x', y')}{\sqrt{(x - x')^2 + (y - y')^2}} dx' dy' \end{aligned} \quad (8)$$

In the formula (8)  $x$  and  $y$  are coordinates of the contact points.

After the insertion of the formula (8) into the formula (5), we will acquire the points in the contact.

$$\begin{aligned} \frac{1 - \mu_1^2}{\pi \cdot E_1} + \frac{1 - \mu_2^2}{\pi \cdot E_2} \cdot \iint_{\Omega} \frac{p(x', y') \cdot dx' dy'}{\sqrt{(x - x')^2 + (y - y')^2}} = \\ = d(x, y) - f_1(x, y) - f_2(x, y) \end{aligned} \quad (9)$$

If we know the contact area and the normal stress distribution in the contact, we can acquire the normal force  $Q$  and the position of the point in which the force acts.

$$Q = \iint_{\Omega} p(x, y) \cdot dx \cdot dy \quad (10)$$

$$x_0 \cdot Q = \iint_{\Omega} x \cdot p(x, y) \cdot dx \cdot dy \quad (11)$$

$$y_0 \cdot Q = \iint_{\Omega} y \cdot p(x, y) \cdot dx \cdot dy \quad (12)$$

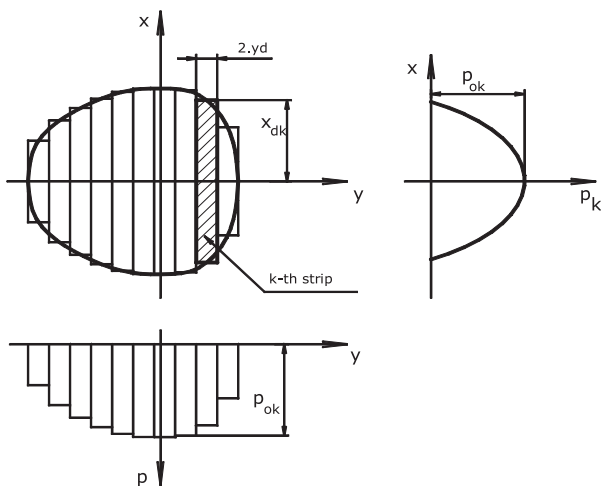


Fig. 5. Contact area division into strips normal pressure distribution in separate strips

The situation is more complicated in the case of the wheel and rail contact because we do not know in advance the acting normal (wheel) force  $Q$  and geometrical profile parameters of the touching bodies (wheel and rail).

The surface deformation  $w(x, y)$  depends on surface functions  $f_1(x, y), f_2(x, y)$  and on the approach  $d$  of both elastic bodies. The unknown distribution of the normal pressure  $p(x, y)$  will be determined on the contact area  $\Omega$  from the sum of the surface deformations of two bodies in the contact  $w(x, y)$  [7, 9]. The base of the strip theory is the presupposition of contact area  $\Omega$  symmetry, around  $y$  axis. The contact area will be divided into  $M$  strips with the length  $2 \cdot x_{dk}$  and width  $2 \cdot y_d$  perpendicular on the  $y$ -axis. It is done for purposes of our numerical calculation. Another presupposition is a semi-elliptic distribution of the normal pressure in the strip direction and a constant in the direction  $y$ .

The results of the calculations - examples

Wheel force:	100 kN	Wheel profile:	R-UIC S1002
Wheel diameter:	460 mm	Rail profile:	S-UIC 60
Lateral displacement: 0 mm.			

$\Omega$  - contact area  
 $\Omega_p$  - penetration area  
 $p_0$  - normal stress

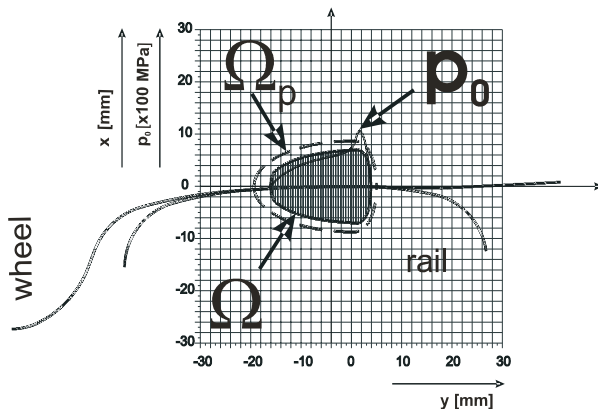


Fig. 6. Contact area and normal stress distribution. Graphs in Fig. 8. to Fig. 13. legend only.

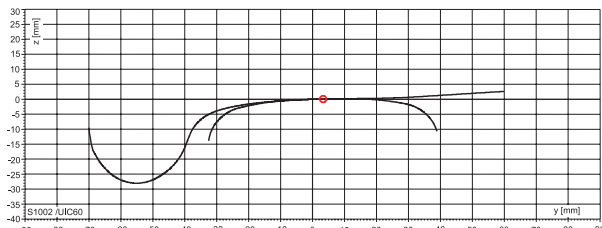


Fig. 7. Right wheel profile of R-UIC1002 and rail head profile S-UIC60 contact point

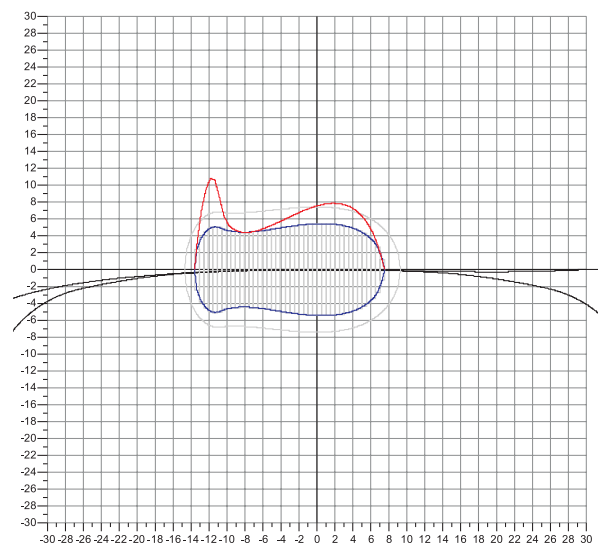


Fig. 8. Wheel lateral displacement state by 0 mm

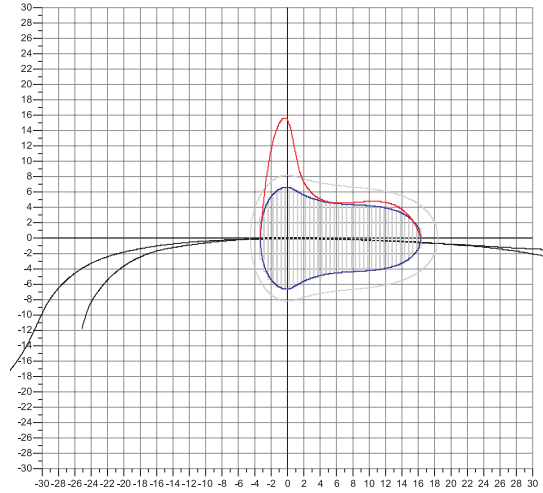


Fig. 9. Wheel lateral displacement state by 1 mm

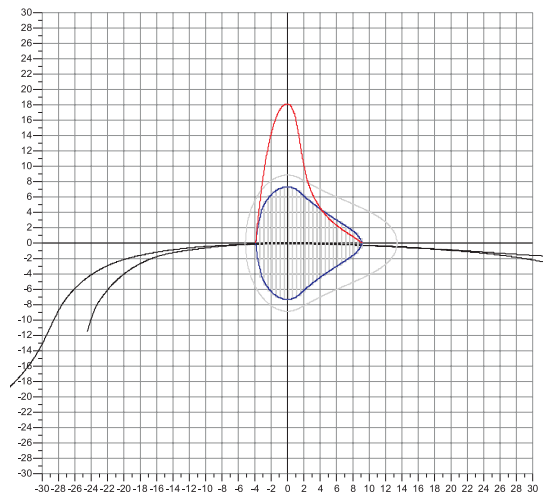


Fig. 10. Wheel lateral displacement state by 2 mm

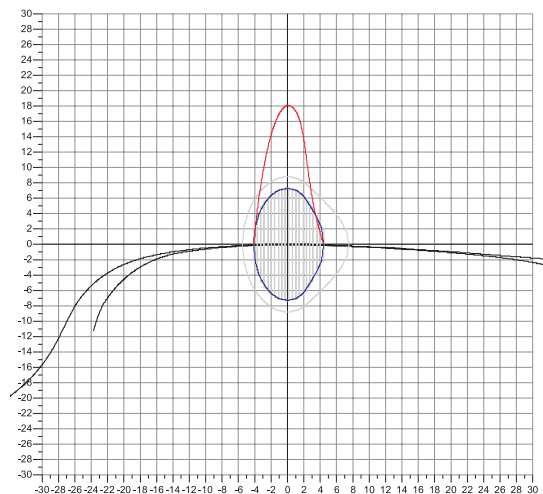


Fig. 11. Wheel lateral displacement state by 3 mm

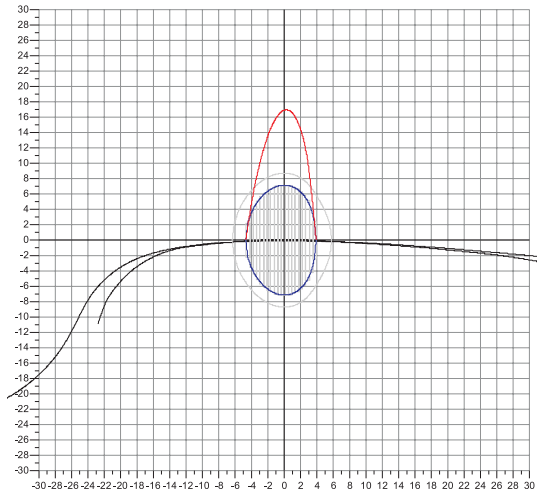


Fig. 12. Wheel lateral displacement state by 4 mm

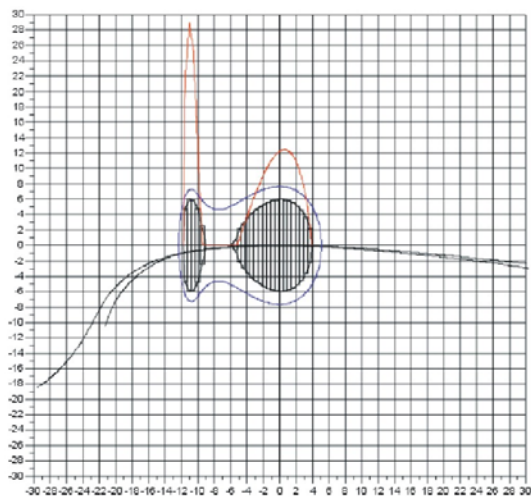


Fig. 13. Wheel lateral displacement state by 5 mm

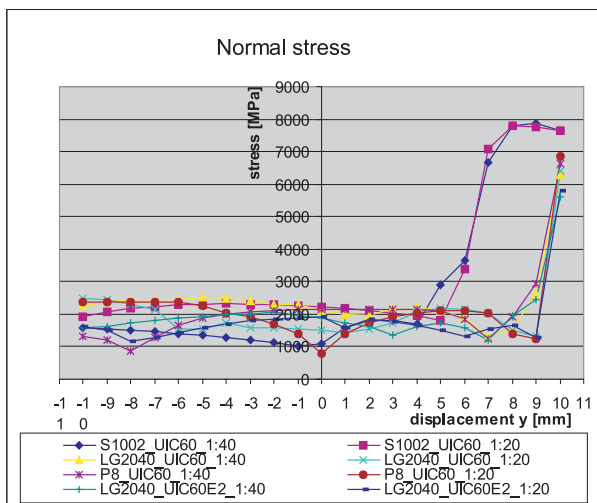


Fig. 14. Normal stress distribution [1]

The calculation is in the field of linear stress and strain and it does not take into account a plastic deformation.

#### 4. Comparison of the calculation results according to the Hertz and strip methods.

Cylinders, angle of axes 90°, loading 100 kN,  $E = 2.1 \times 10^6$  MPa, Poisson's ratio 0.277.

Profiles: wheel S1002, rail UIC 60

Cylinders, angle of axes 90°, loading force 100 kN,  $E = 2.1 \times 10^6$  MPa, Poisson's ratio 0.277

Tab. 1.

Wheel	Rail	Hertz method			
R1	R2	A	b	area	$P_{max}$
300	300	11.65	11.65	106.7	1406.2
460	300	14.47	10.89	123.7	1212.5
625	300	16.91	10.42	138.39	1083.9
		Strip method			
R1	R2	X (a)	Y (b)	area	$P_{max}$
300	300	11.33	11.25	101.2	1582
460	300	13.66	10.61	115.1	1402
625	300	15.54	10.14	125.8	1289

Curvatures radii of a S1002 wheel profile and a UIC60 rail profile in the contact point at lateral movement of a wheel profile.

Tab. 2.

Nr.	shift	Wheel curvature	Rail curvature
1	-10	820.44	299.99
2	-9	1084.3	300.01
3	-8	1640.84	300.04
4	-7	3402.45	300.06
5	-6	30184.75	300.06
6	-5	2652.18	300.05
7	-4	1384.58	300.02
8	-3	899.57	300
9	-2	659.38	299.96
10	-1	508.64	299.97
11	0	392.26	300
12	1	199.82	93.87
13	2	181.21	81.68
14	3	161.68	80.14
15	4	139.91	80
16	5	113.78	79.99
17	6	21.01	13.72
18	7	108.78	27.63
19	8	40.39	23
20	9	26.61	17.01
21	10	21.09	14.9

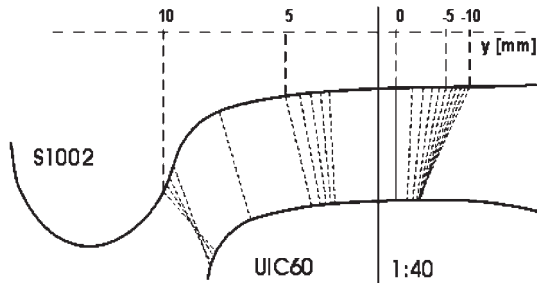


Fig. 15. Contact points at lateral movement of a wheel profile

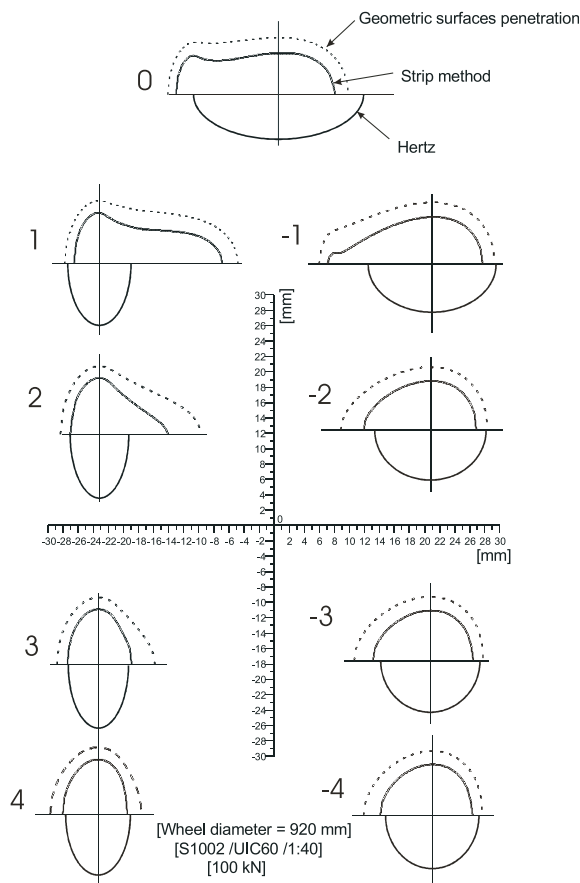


Fig. 16. Shape and size of contact areas for S1002/UIC60/1:40/Q100kN/D920 movement +/- 4 mm [1]

Many researchers have worked in the field of analysis of shape and size of the wheel and rail contact area. The main axis in the case of the strip method is the longer distance of longitudinal and transversal border lines.

More or less according to the main interest of their professional specialization, they used either apparatus of derived formulae (Hertz, Kalker, Knothe, LeThe) or procedures based on the finite element method (Wriggers).

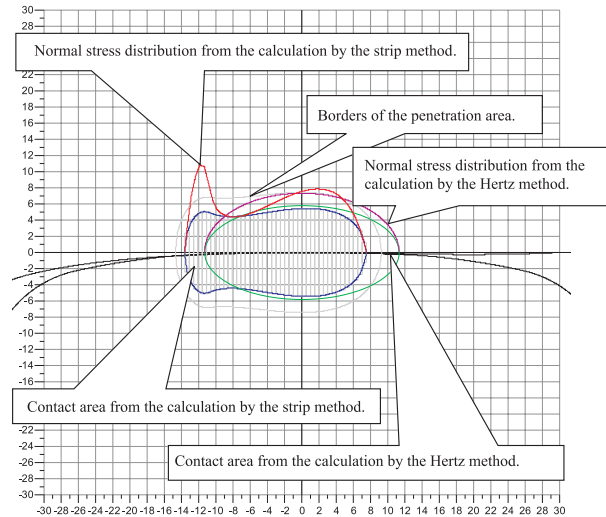


Fig. 17. Contact area calculated by the Hertz and strip method and normal stresses at zero movement of a wheel profile corresponding to the methods

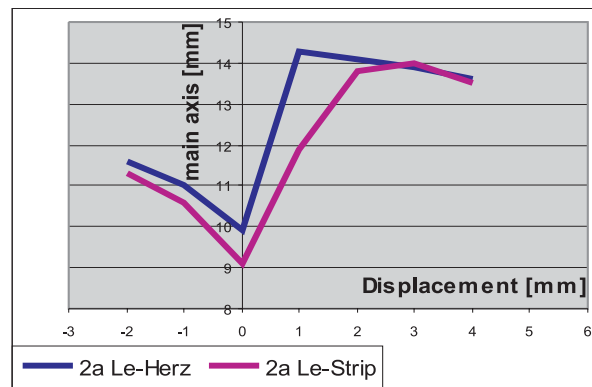


Fig. 18. The principal axis length of contact ellipses calculated by the Hertz method and by the strip method Le The Hung [5]

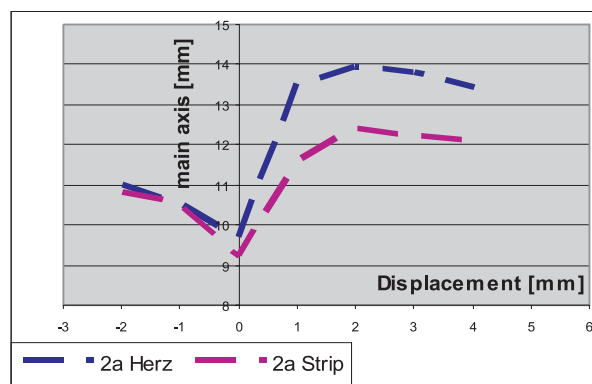


Fig. 19. The principal axis length of contact ellipses calculated by the Hertz method and by the strip method Lack-Gerlici [1]

In the previous text the calculations were done on the base of the Hertz and strip method in the field of the linear theory of stress and strain.

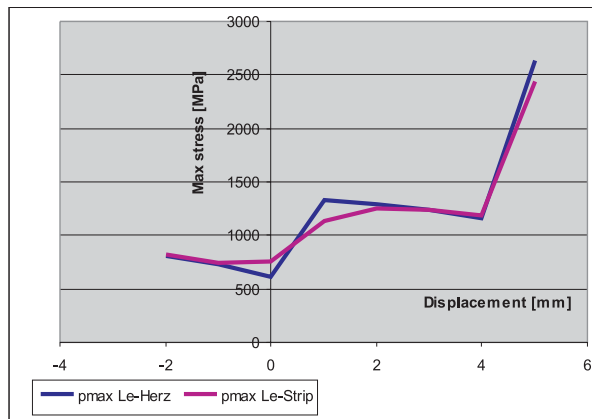


Fig. 20. Size of maximum stress in the contact area calculated by the Hertz method and strip method - Le The Hung [5]

The reason why it is necessary to work in the field is crucial for the choice of research methods and the evaluation of geometrical and force phenomena which arise at the rail and wheel contact.

The shape and size of the contact area and the normal stress distribution is the core of the interest in this case.

A plausible analytical determination of the normal stress should follow after an analysis of geometric characteristics and should proceed for a possible analysis of the rail and wheel contact, or ride mechanics, or the whole train dynamics.

The aim of the comparison is to prove correctness or better to say acceptance of an analytical research method of the problems as a base for further analysis. In this case, it is not possible to mark all the calculations as correct or incorrect. More aspects influence differences in numerical expression. The greatest influence has the following: the chosen method of the numerical calculation, the chosen accuracy of the calculation, material characteristics, geometrical characteristics (wheel and rail profile shape, rails slope, nominal wheel radius). In Figs. 18 to 21 there is a comparison of calculation results of the main and side contact ellipses length and a normal stress maximum with the help of the Hertz and strip methods. If we look at the results shown in the figure it is possible to judge the suitability (probably correctness) of the method itself and the rate of the results identity which have been acquired via the application of the same method by various researchers. In the first case (suitability and usability of the method), it has been proved that the methods provide more identical results in the case which is more suitable for them mainly from the point of view of geometrical shapes of the contact bodies.

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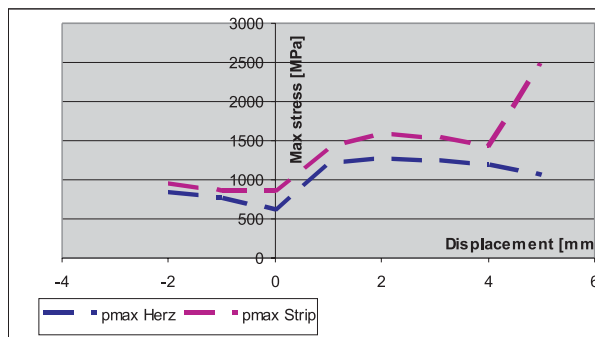


Fig. 21. Maximum stress size in the contact area calculated by the Hertz method and strip method - Lack-Gerlici

The Hertz method is for sure approved and acceptable for further analysis where we require speed of data numerical elaboration, uniformity in the calculation procedure (special algorithm is quite good available) for geometrically more simple bodies shapes.

The strip method extends possibilities of calculations by an analysis of non-elliptical contact as well. Another procedure of the calculation enables to find out the normal stress distribution which can be different from the Hertz procedure. The results published in [7] were used for the graphs in Figs. 18-21. On the base of entry data from the same literature: UIC60, S1002, 1:40, 60kN,  $E = 2.1 \times 10^6$  MPa, Poisson's ratio = 0.277, they were calculated according to the procedures mentioned in the paper.

## 5. Conclusions

In the paper, the well known calculation procedure by Hertz, which results in contact ellipses, was used for searching the shape and size of the contact area as well as for searching the contact normal stress. The ellipses do not always provide a sufficient idea about a real contact area and from it arising normal stress. The Hertz method was compared with a new version of the strip method, which provides more precise results. In both methods, we work with simplified presuppositions which deal with an elastic model. The methods and their results were used for judging the influence of various profile shapes of wheels and rails on the shape and size of the contact area and normal stress. The results proved a significant influence of contact bodies shapes (wheel and rail) on the shape and size of the contact area as well as on the stress size and distribution over the contact area. The contact was understood as quasi-static not taking into account influences and consequences of a wheel rolling.

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