

# CALCULATION OF THE EQUIVALENT CONICITY FUNCTION OF THE RAILWAY WHEELSET TREAD PROFILE AT THE DELTA R FUNCTION WITH A NEGATIVE SLOPE

The rail-wheel interface is fundamental to explain the dynamic running behaviour of a railway vehicle. It must therefore be understood and among the parameters by which it is characterised, the one called "equivalent conicity" plays an essential role since it allows the satisfactory appreciation of the wheel-rail contact on tangent track and on large-radius curves.

By definition, the equivalent conicity is equal to the tangent of the cone angle  $\text{tg } \gamma_e$  of a wheelset with coned wheels whose lateral movement has the same kinematic wavelength as the given wheelset.

In order to be able to compare the results obtained by different railways, both as to the value of the equivalent conicity and as to the results where that parameter plays an important role, it is necessary for the equivalent conicity to be calculated according to the same principles.

Methods, which determine ways of the calculation, are mentioned in the UIC leaflet, but it does not define either limits of accepted values of the equivalent conicity or the shape profiles, thanks to which the conicity could be reached.

The equivalent conicity is calculated either by the application of the Klingel formula or with the help of the linear regression of the  $\Delta r$  function on the interval of the double amplitude of the wavelength of a periodical movement of the middle of a wheelset. The process of the calculation in the leaflet is determined for a standard case of the  $\Delta r$  function shape, which crosses the y-axis in one point in the graph of dependence of the  $\Delta r$  function on the lateral displacement. In practice there are cases when the  $\Delta r$  function crosses the y-axis in more than one point.

The paper deals with the possibility of the calculation of the equivalent conicity with the help of the Klingel formula in the case when  $\Delta r$  function crosses the y-axis in three points.

The leaflet describes such a case as a  $\Delta r$  function course with a negative slope. In this case not one, but three curves will be the results. Each of them will determine an equivalent conicity for another interval of validity (amplitude of oscillation of the middle of the wheelset). According to the initial conditions (amplitude of oscillation and position of the wheelset on the rail), the wheelset can have for some amplitudes two possible trajectories or none in the case of a negative conicity.

## 1. Introduction

Forces, thanks to which the wheelset comes back to a centred position, act on the railway wheelset after the deflection of its middle from the railway axis in the lateral direction. The wheelset performs at the same time a forward movement in the track direction. The result of the simultaneous movement, both in the progressive direction and in the lateral direction, is a theoretically regular wavy movement of the wheelset which is represented by its centre.

The conicity of tread profiles of railway wheels is the reason why the wheelset has a tendency to come back in the lateral direction from maximum deviations into the middle of a track. This happens when driving even after a lateral deviation from the centred position against the rail axis. It is important to determinate courses of geometrical characteristics of profile contacts of a railway wheel and a rail for the evaluation of the tendency rate.

## 2. Geometrical characteristics

Geometrical characteristics of a wheel and a rail contact are the following [2, 3]:

*Contact points* determine the position of contact points at the lateral displacement of the wheel profile on the rail profile.

*Delta -r ( $\Delta r$  function)* is a difference between of an instant diameter of running of one wheel of a wheelset and instant diameter of running of other wheel of a wheelset at the lateral movement of a wheelset on a track.

*Tangent Gamma function* is a difference between the value of the normal line tangent angle of the tangential contact area in the contact point and vertical axes of a track. It determines the rate of binding intensity, which comes back the wheelset after its lateral deviation into a centred equilibrium position on the track.

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*Effective conicity* is a reference conicity only of a cone profile of a wheel of a wheelset which the wheelset should possess to perform the same rotational movement without slips on the track as the real (measured) wheelset.

*Equivalent conicity* is a reference conicity which equals the curving profile of a wheel when taking into consideration the wavelength and amplitude of a coned wheel movement.

The angle of attack and the angle of tilt of the wheelset, which originated by a different instant diameter of rolling, influences the size and shape of geometrical characteristics.

In the following text, let us suppose that the wheelset moves on the track under the angle of attack which equals zero grade and we will not take into consideration the influence of the angle of tilt on the shape of geometrical characteristics. Each of the above mentioned geometrical characteristics shows a measure of other condition of the geometrical contact, the position of contact points and the  $\Delta r$  function shape are the basis for the determination of other characteristics. The article concentrates on the determination of the equivalent conicity function. The methods of the calculation of the equivalent conicity function are determined by the UIC leaflet [1]. The basic procedure of the calculation of the equivalent conicity presupposes a standard shape of the  $\Delta r$  function course which crosses the horizontal axis in one point.

When there are wheel profiles with modified geometry, for example by wearing of the wheel tread, it can happen that the  $\Delta r$  function crosses the horizontal axis in three points. The leaflet describes this case as a characteristic with a negative slope and from the point of view of the solution procedure it enables a certain variability.

### 3. Equivalent conicity

By definition, the equivalent conicity is equal to the tangent of the cone angle  $\text{tg } \gamma_e$  of a wheelset with coned wheels whose lateral movement has the same kinematic wavelength as the given wheelset (on tangent track and on large-curve radius curves) [1].

If a coned tread profile of a wheel is used, and we do not take into consideration the slip between a wheel and a rail, the shape of the wave will have periodical oscillating course.

If we know the wheel diameters and the track gauge (distance of contact points), it is possible to state at the periodical oscillating movement the following [4]:

The wavelength does not depend on the amplitude and it is set only by the conicity of the tread profile of a wheel. The maximum amplitude is defined only by initial conditions.

Supposing that there is no slip between a wheel and a track, maximum amplitudes of waves following each other do not change.

In reality forces act on the wheelset moving on a track. The forces will lead to certain slips between the wheel and the rail they are determined by the relation friction - adhesion coefficient.

As a result of this influence, the amplitudes of waves following each other change in this way:

- the amplitudes of the waves decrease → the wheelset moves “in a stable way”
- the amplitudes of the waves increase → the wheelset moves “in an instable way”,
- the amplitudes of the waves do not change → the wheelset moves in limits - cycles.

Determining the equivalent conicity course

The equivalent conicity is found by a *two-step computation* as follows [1]:

#### 1. Integration of the differential equation

$$\frac{d^2y}{dx^2} + \frac{\Delta r}{2 \cdot s \cdot r_0} = 0 \quad (1)$$

using the previously set characteristic  $\Delta r = f(y)$  together with the following conditions:

$$y = y_0 \text{ for } x = 0 \quad (2)$$

$$\frac{dy}{dx} = 0 \text{ for } x = 0 \quad (3)$$

- where:  $x$  displacement of the wheelset of the longitudinal direction of the track  
 $y$  displacement of the wheelset of the lateral direction of the track  
 $s$  half taping line distance  
 $r_0$  radius of the wheels when the wheelset is centred on the track  
 $\Delta r$  difference of the rolling radius between right hand and left hand wheels.

The integration, based on the initial amplitude  $y_0$ , leads to the periodical movement of the wheelset with an amplitude (between two peaks) with the size  $2\hat{y}$  and wavelength  $\lambda$ .

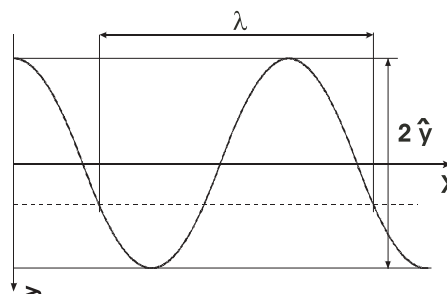


Fig. 1  $y = F(x)$  function

### 2. Calculation of the conicity

From the actual movement of the wheelset the conicity is computed for the amplitude  $\hat{y}$  either by applying the Klingel formula

$$\operatorname{tg} \gamma_e = \left(\frac{\pi}{\lambda}\right)^2 \cdot 4 \cdot r_0 \cdot s \quad (4)$$

where  $\operatorname{tg} \gamma_e$  equivalent conicity  
 $\lambda$  wavelength  
 $\hat{y}$  wave amplitude

or by applying a least squares type linear regression to the portion of the  $\Delta r = f(y)$  characteristic within the  $\hat{y}$  interval. The slope of this regression is equal to  $2 \cdot \operatorname{tg} \gamma_e$ .

The procedure of the calculation of the equivalent conicity function course differs from the point of view of number of intersections of the Delta  $r$  function  $y$ -axis course. For practical usage that is for the equivalent conicity determination, the method of the Klingel formula application was used.

The wheelset's movement on the track can be formalised on the basis of the angle of attack  $\Psi$ .

$$\Psi = \frac{\dot{y}}{\dot{x}} = \frac{dy}{dx} \quad (5)$$

$d\Psi$  increment of the angle of attack  
 $ds$  length of the track arc which equals the size of the angle  $d\Psi$   
 $R$  local radius of the wheel path

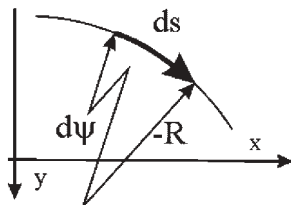


Fig. 2 Track increment

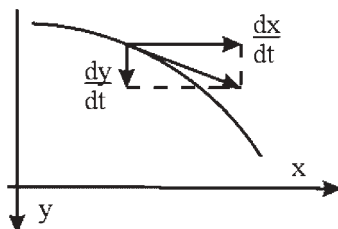


Fig. 3 Velocity vector and its components

$$ds = -R \cdot d\Psi \quad (6)$$

for small angles  $\Psi$ ,  $ds \cong dx$

$$dx = -R d\Psi \quad (7)$$

and taking into account (1), this yields

$$\Psi = \frac{dy}{dx} = -\frac{dy}{R d\Psi}, \text{ hence } \Psi d\Psi = -\frac{dy}{R} \quad (8)$$

$$\frac{R}{2 \cdot s} = \frac{r_1 + r_2}{r_1 - r_2} \quad (9)$$

where  $r_1$  right wheel diameter  
 $r_2$  left wheel diameter

$$r_0 = \frac{r_1 + r_2}{2} \quad (10)$$

where  $r_0$  nominal radius of each wheel at centred position of wheelset in a track

$$\frac{r_1 - r_2}{2 \cdot s} = \Delta r \quad \Delta r \text{ (Delta-r) tapping line distance}$$

Thus:

$$R = \frac{r_0 \cdot 2 \cdot s}{\Delta r} \quad (11)$$

And by replacing  $R$  in (8), we get

$$\Psi d\Psi = \frac{-\Delta r \cdot dy}{r_0 \cdot 2 \cdot s} \quad (12)$$

giving, by integration,

$$\frac{\Psi^2}{2} = \frac{1}{2 \cdot s \cdot r_0} \left[ \int \Delta r dy + C \right] \quad (13)$$

This integration serves to obtain the movement of the wheelset on the track and, in particular, the path corresponding to half of the wavelength starting from  $y_{emin}$  (for which  $\Psi_{emin} = 0$ ) and going up to  $y_{emax}$  (for which  $\Psi_{emax} = 0$ ).

The integral  $\int \Delta r \cdot dy$  need only be calculated once for sufficiently large amplitudes to be able to cover the domain that will be needed for further assessment.

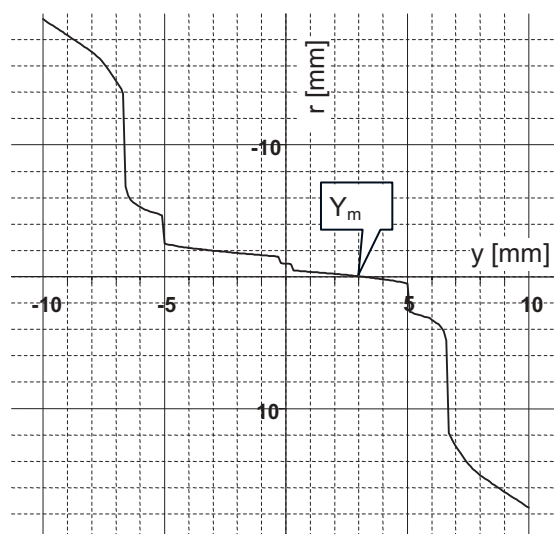


Fig. 4  $\Delta r$  function with one intersection of  $y$ -axis

The wheelset's movement on the track is then obtained with the help of the following integration:

$$dx = \frac{1}{\Psi} dy \text{ or } x = \int \frac{dy}{\Psi} \quad (14)$$

The procedure of the solution will be chosen on the base of  $y$ -axis intersections number.

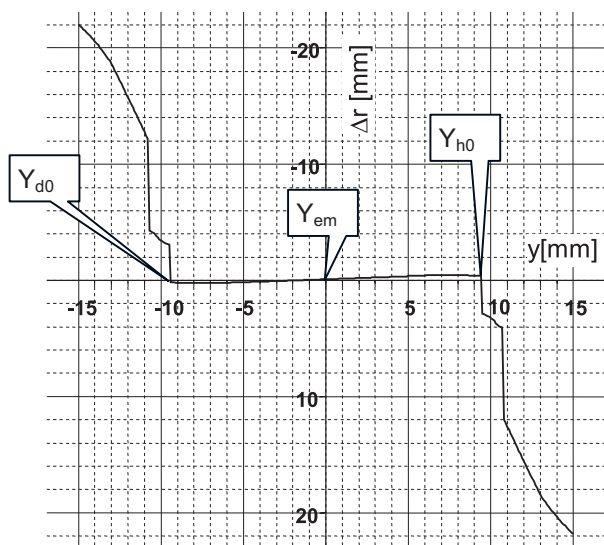


Fig. 5  $\Delta r$  function with three intersections of  $y$ -axis

### 3. $\Delta r$ function with one intersection of the $y$ -axis

In practice the method of the Klingel formula application for one intersection of the  $y$ -axis is used in the following way.

1. Based on the function  $\Delta r = r_1 - r_2 = f(y)$ , find the value of  $Y_{em}$  that corresponds to  $\Delta r = 0$ . (The example of  $\Delta r$  function with one intersection is in Fig. 4).
2. Calculate the function  $S(y) = \int \Delta r \cdot dy$ , starting from  $y_{em} = Y_{em}$  in steps of  $dy = +0.1$  mm to  $y$  and from  $y_{em} = Y_{em}$  in steps of  $dy = -0.1$  mm do  $-y$ .

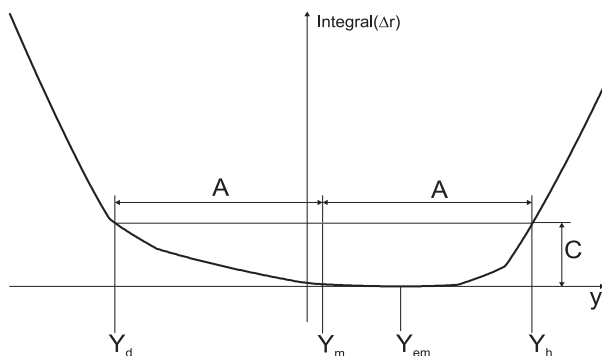


Fig. 6 Graph of the integral  $\Delta r$  function with one  $y$ -axis intersection

3. Determine the corresponding amplitudes  $y_{emin} = Y_d$  a  $y_{emax} = Y_h$  and calculate the mean lateral movement  $\hat{y} = Y_m$ .
4. Find the functions  $y_{emin} = f(\hat{y})$  a  $y_{emax} = f(\hat{y})$  which allow to determine, for given lateral movement of the wheelset  $2\hat{y} = 2A$ , corresponding minimum and maximum amplitudes  $y_{emin}$  and  $y_{emax}$ .
5. Compute the equivalent conicity  $tg\gamma_e$  for a given movement  $\hat{y} = A$ : find the constant  $C$  of equation (13), such  $\Psi_{emin}$  for the corresponding  $y_{emin}$ , calculate the angle of attack  $\Psi$  by integrating equation (12) to give

$$\Psi = \sqrt{\frac{-1}{r_0 \cdot s} \left[ \int \Delta r \cdot dy + C \right]} \quad (15)$$

In steps of  $dy = 0.1$  mm.

Calculate the abscissa of the wheelset movement  $x = f(y) = \int \frac{dy}{\Psi}$  between  $y_{emin}$  and  $y_{emax}$ , which allows to find the wavelength  $\lambda$  of the wheelset's kinematic motion.

In most cases the integration cannot be done in only one step in the range from  $y_{emin}$  to  $y_{emax}$ .

Therefore the wavelength  $x$  must be calculated by adding up,  $dy = \frac{dy}{\Psi}$  whereby the step of  $dy$  should be lower than 0.1 mm.

Calculate the equivalent conicity, applying the Klingel formula.

$$tg\gamma_e = \left( \frac{\pi}{\lambda} \right)^2 \cdot 4r_0 \cdot s \quad (16)$$

6. Determine the function  $tg\gamma_e = f(\hat{y})$  by applying Step 5. For  $\hat{y}$  amplitudes starting up to the maximum permitted by the  $\Delta r = f(y)$  characteristic, with a maximum step  $\hat{y} = 0.5$ mm.

### 4. $\Delta r$ function with three intersections of the $y$ -axis (with a negative slope)

In practice there are cases when  $\Delta r$  function crosses the  $y$ -axis in more than one point. (Fig. 5). The presentation of profiles shape and the distribution of contact points is in Fig.7. In this case, the wheelset according to initial conditions oscillates around one of the positions. There are three curves which describe the situation in the graph in Fig. 11.

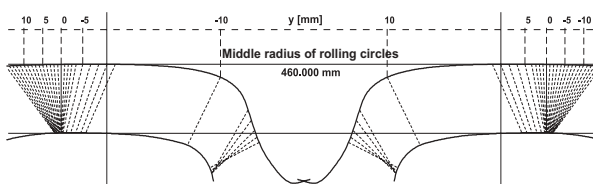


Fig. 7 Profiles shape and contact points distribution

When there are three intersections of the  $y$  axis, the graph of the  $\Delta r$  functions has a shape, which is shown in Fig. 8. Instead of a simple shape of the integral solution (around one intersection  $Y_m$ , Fig. 6), the geometrical shape of the integral solution consists of calculations around the first and third intersections of the  $\Delta r$  function through the  $y$ -axis.

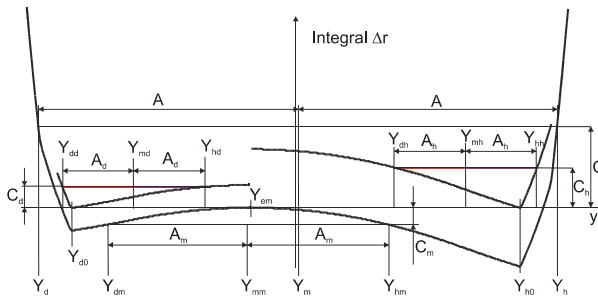


Fig. 8 Graph of the  $\Delta r$  integrals with three  $y$ -axis intersections

## 5. Calculation of $\Delta r$ function integrals

When  $\Delta r = f(y)$  characteristic has three intersections of the  $y$ -axis (defined as a characteristic having a negative slope), a slightly different procedure of equivalent conicity calculation is necessary.

Analysis of the  $\Delta r$  function shape,

- number of  $y$ -axis intersections,
- setting of the  $Y_{em}$  value,
- when there are three intersections setting of the  $Y_{d0}$  and  $Y_{h0}$  values as well,
- calculation of the curve of the  $\Delta r$  function integral,
- calculation of the integral on the left from  $Y_{em}$ ,
- calculation of the integral on the right from  $Y_{em}$ ,
- when there are three intersections of the  $y$ -axis,
  - calculation of the integral on the left from  $Y_{d0}$ ,
  - calculation of the integral on the right from  $Y_{d0}$ ,
  - calculation of the integral on the left from  $Y_{h0}$ ,
  - calculation of the integral on the right from  $Y_{h0}$ .

When the integrals for separate  $Y$  are calculated it is possible to gain other variables.

Calculation of the equivalent conicity course and mean value.

$$r = \frac{r_1 + r_2}{2} \quad (17)$$

where:  $r_1$  right wheel radius  
 $r_2$  left wheel radius  
 $r$  mean diameter of the wheels of the wheelset

## 6. For oscillation around $Y_{em}$

From  $y_0 = y_{0 \min}$  to  $y_{0 \max}$  with the step  $\Delta y_0$

1. Determination of the  $C, Y_d, Y_h, Y_m$  constants for the  $y_0$  amplitude under the condition  $Abs(y_0 - A) < 1.10^{-4}$

For the angle of wheelset yawing (angle of attack)

$$\psi = \sqrt{\frac{C - [\Delta r](y)}{s \cdot r}} \quad (18)$$

2. Determination of the function course.

$$\frac{1}{\psi} = f(y) \quad (19)$$

3. Determination of the mean value of the function  $\frac{1}{\psi}$

$$\overline{\left(\frac{1}{\psi}\right)} = \frac{\int_{y_d}^{y_h} \frac{1}{\psi}}{2 \cdot y_0} \quad (20)$$

Or for a negative conicity

$$\overline{\left(\frac{1}{\psi}\right)} = \frac{\int_{y_{dm}}^{y_{hm}} \frac{1}{\psi}}{2 \cdot y_0} \quad (21)$$

4. Determination of the wavelength of the wheelset movement

$$\lambda_0 = 4 \cdot \overline{\left(\frac{1}{\psi}\right)} \cdot y_0 \quad (22)$$

5.  $\lambda$  = calculation of the wheelset movement ( $s, r_1, r_2, \Delta r, y_0, y_m, \lambda_0$ )

If  $\lambda > 0, Y_m[y_0] = y_m$

$$K[y_0] = \left(\frac{2 \cdot \pi}{\lambda}\right)^2 \cdot s \cdot r \quad (23)$$

When there are three intersections of the  $\Delta r$  function with  $y$ -axis

$\lambda$  = Calculation of the wheelset movement ( $-s, r_1, r_2, \Delta r, y_0, y_m, -\lambda_0$ )

If  $\lambda > 0, Y_m[y_0] = y_m$

$$K[y_0] = \left(\frac{2 \cdot \pi}{\lambda}\right)^2 \cdot s \cdot r \quad (24)$$

- When there are three intersections of the  $y$ -axis, for the oscillation around  $Y_{d0}$  and  $Y_{h0}$  the calculation is done in the same way as for the oscillation around  $Y_{em}$  except for the negative conicity.

## 7. Presentation of the calculation results

$K$  vector of the equivalent conicity

$Y_m$  vector of the mean value

$K_d$  vector of the equivalent conicity for the oscillation around  $Y_{d0}$

$Y_{md}$  vector of mean value for the oscillation around  $Y_{d0}$

$K_h$  vector of the equivalent conicity for the oscillation around  $Y_{h0}$

$Y_{mh}$  vector of mean value for the oscillation around  $Y_{h0}$

One curve of the dependence on the amplitude of the lateral movement represents the graph of the equivalent conicity. The position of the axis around which the middle of the wheelset oscillates

lates ( $Y_m$ ) is unambiguously changed according to size of the amplitude. (Fig. 9)

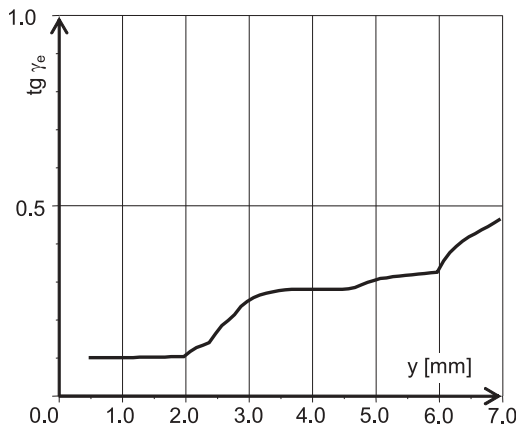


Fig. 8 Graph of equivalent conicity of the  $\Delta r$  function with one  $y$ -axis intersection

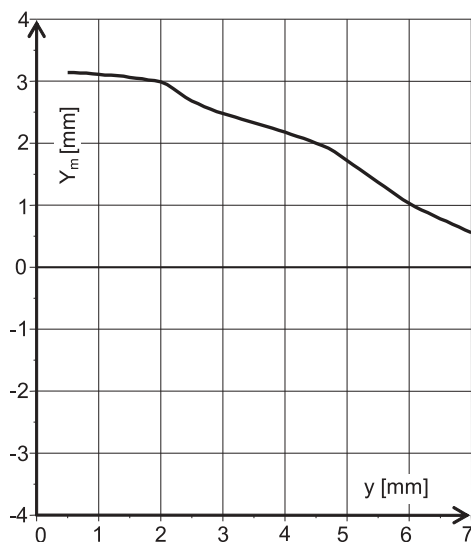


Fig. 9 Graph of mean  $Y_m$  for  $\Delta r$  function with one  $y$ -axis intersection

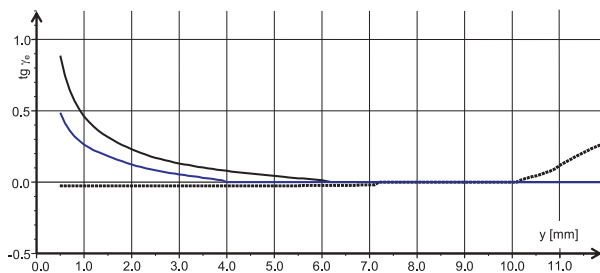


Fig. 10 Graph of the equivalent conicity function for  $\Delta r$  function with three  $y$ -axis intersections

It is possible to determine the equivalent conicity (Fig. 10) from several curves as a result of the  $\Delta r$  function integral shape.

Two curves above the zero axis equal two possible periodical movements. The curve below the axis represents a theoretically non-stable area which equals the equivalent conicity non-leading to periodical movement.

The oscillation of the wheelset in the frame of amplitudes of approx. 6–10 mm is excluded in this specific case. When the amplitudes are smaller, it oscillates according to some of two local oscillation axes (Fig. 12) or when the amplitude is about 10.3 mm it oscillates only in one possible way (the third “middle” intersection  $\Delta r \sim Y_{em}$ ) (Fig. 14.)

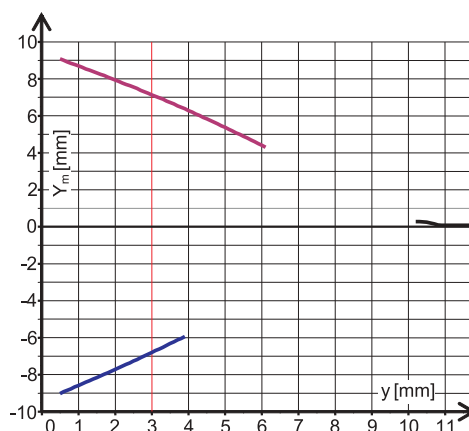


Fig. 11 Graph of mean  $Y_m$  ( $Y_{md}$ ,  $Y_{mh}$ ) for  $\Delta r$  function with three  $y$ -axis intersections

The shape of the trajectory of the wheelset centre arises from the size of the amplitude and from the position around which the wheelset centre oscillates. Both possibilities of the path of the wheelset centre for the amplitude of 3 mm are in Fig. 12.

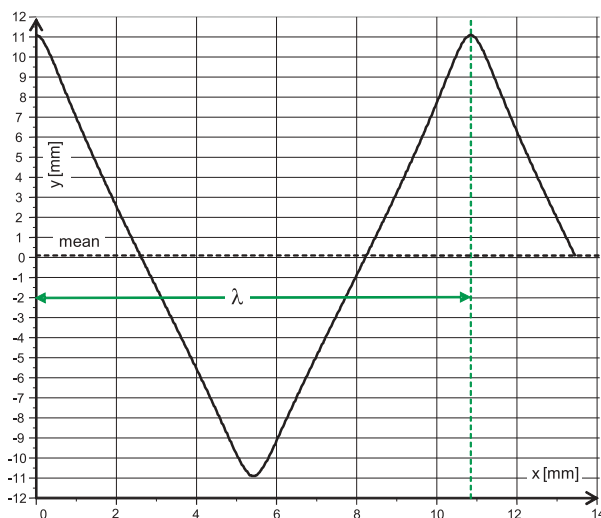


Fig. 14 Movement of a wheelset for the  $\Delta r$  function with three  $y$ -axis int. and  $y_0 = 11$  mm

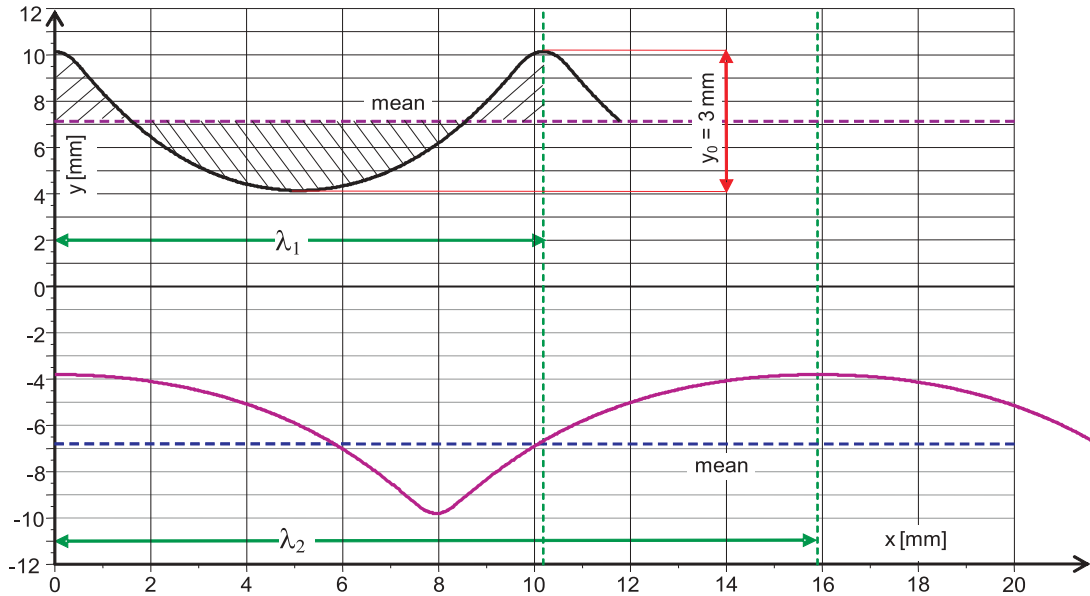


Fig. 12 Movement of a wheelset for the  $\Delta r$  function with three  $y$ -axis int. and  $y_0 = 3$  mm

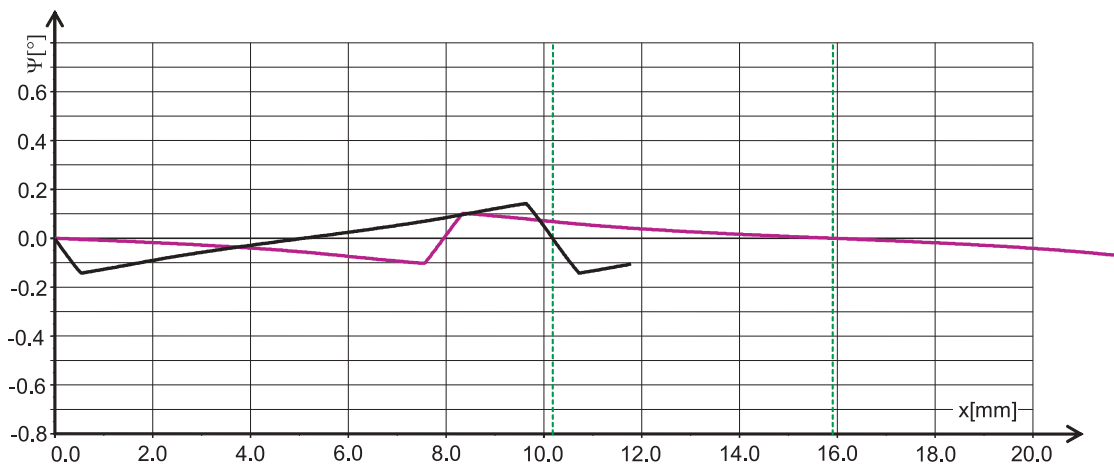


Fig. 13 Turning of a wheelset for the  $\Delta r$  function with three  $y$ -axis intersects and  $y_0 = 3$  mm

The turning of the wheelset (angle of attack) in degrees at the periodical movement of the wheelset centre with the amplitude of 3-mm is shown in Fig. 13. Two curves build the graph, each of the curves is valid for other position of the centre around which the wheelset oscillates.

There is only one trajectory of the potential movement of the wheelset centre for the amplitude of 11-mm in Fig. 15.

The course of the wheelset yawing (angle of attack) in degrees at the periodical movement of the wheelset centre with the amplitude of 11 mm (shown in Fig. 15) is only one curve. The curve course of the wheelset yawing is closely connected with the shape of the  $\Delta r$  functions integrals.

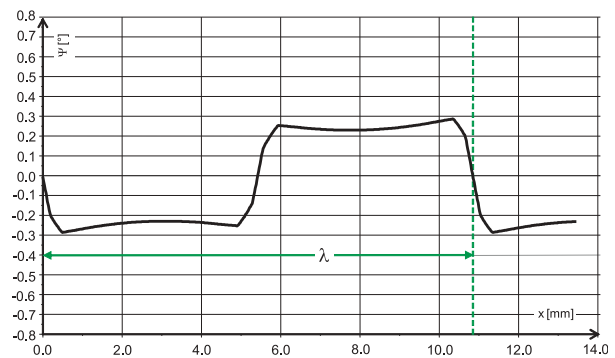


Fig. 15 Turning of a wheelset for the  $\Delta r$  function with three  $y$ -axis int. and  $y_0 = 11$  mm

## 7. Conclusion

When evaluating the equivalent conicity and other geometric characteristics of the wheelset, it is necessary to take into account real shapes of the rails and wheel treads of both the wheels of the wheelset together with radii of the wheels.

In some cases the difference of the radii of the rolling of the wheels ( $\Delta r$ -function) obtains a negative slope. The fact will be realised in more than one intersection of the delta  $r$  function of the  $y$ -axis in the graph  $\Delta r = f(y)$ . The rolling radius of the wheel approaching the rail is decreasing this time while the radius of the drawing apart wheels from the rail is increasing.

The result of this geometric characteristic is that the wheelset can move in a wavy movement not around one but around more

different positions. When the amplitudes are relatively small, it oscillates around one of two possible positions and it starts to oscillate only around one position only when it exceeds the limit value of the amplitude. The change is jumping, at a certain interval of amplitudes the wheelset cannot oscillate.

As the evaluation of the equivalent conicity is based on the analysis of the wavy movement of the wheelset on the track, we achieve three different courses of the equivalent conicity functions. The fact requests a special procedure of the evaluation of the equivalent conicity which is mentioned in the paper together with application examples.

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