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# POPÍSANIE KREHKÉHO POŠKODENIA BETÓNU

## DESCRIPTION OF BRITTLE DAMAGE IN CONCRETE

*Prezentovaný je jednoduchý model vývoja krehkého poškodenia betónu. Predpokladá sa izotropický vývoj poškodenia a úroveň poškodenia je popísaná skalárnym parametrom. Formulované sú fyzikálne rovnice problému.*

*A simple model of the brittle damage evolution in concrete is presented. The damage evolution is assumed as an isotropic one and the level of damages in material is described by the scalar parameter. The physical equations for this case are formulated.*

### 1. Introduction

The processes of brittle and viscous deformations take place together especially in typical building capillary-porous materials as concrete, ceramics and gypsum [2]. An assumption, that brittle damages (microcracks) occur immediately after imposing a load on the body unlike viscous deformations which are controlled mainly by the diffusion of moisture in capillary tubes of material, is a simple way to describe this phenomenon. The global description of the considered process and its influence on the level of stresses and strains in a viscoelastic body is the aim of this paper.

### 2. Stress and strain transformation

The classical stress tensor transformation

$$\sigma_{ij} = O_{ik}\, O_{jl}\, \sigma_{kl}\,, \tag{1}$$

where $\sigma_{ij}$ – stress tensor, $O_{ik}$ – transformation tensor, $i, j, k, l = 1, 2, 3$, in the body with structure damages must be modified in accordance with the limitation that principal tensile stresses are the main reason of damage evolution [2, 3, 6]. That is why the stress tensor transformation must be divided into two phases (Fig. 1). First, the stress tensor must be transformed to its principal directions and next to the co-ordinate system which we want to analyse the process in [5]. We introduce here a new operation realised in the co-ordinate system compatible with the principal directions of stress tensor

$$\langle\sigma_p\rangle = \frac{1}{2}(\sigma_p + |\sigma_p|)\,, \tag{2}$$

where: $\sigma_p$ – principal stresses, $p = 1, 2, 3$,

which allows us to write the transformation formula

$$\sigma^+_{ij} = O_{ik}\, O_{jl}\, [O^+_{kr}\, O^+_{ls}]\sigma_{rs} = O_{ik}\, O_{jl}\, \langle\sigma_{kl}\rangle = P^+_{ijrs}\sigma_{rs}\,, \tag{3}$$

where

$$P^+_{ijrs} = O_{ik}\, O_{jl}\, [O^+_{kr}\, O^+_{ls}]\,. \tag{4}$$

The same transformation formula is applied to the strain tensor

$$\varepsilon^+_{ij} = P^+_{ijrs}\, \varepsilon_{rs}\,, \tag{5}$$

where $\varepsilon_{ij}$ – strain tensor.

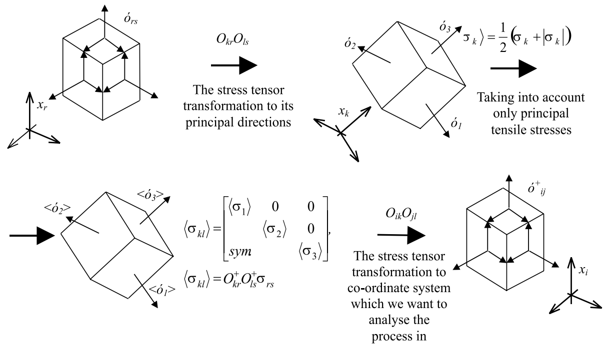


*Fig. 1. Graphical interpretation of the stress tensor transformation divided into two phases.*

### 2. Physical equations

The general form of the physical equation connecting the stress tensor with the strain tensor in an elastic material with structure damages can be formulated as follows

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$$\sigma_{ij} = [1 - \omega]E_{ijkl}\varepsilon_{kl} = E_{ijkl}\varepsilon_{kl} - \omega E_{ijkl}\varepsilon_{kl} = \sigma^0_{ij} - \sigma^D_{ij}, \quad (6)$$

where: $E_{ijkl}$ – tensor containing the constants which describe stiffness of undamaged material, $\sigma^0_{ij}$ – total stress tensor, $\sigma^D_{ij}$ – part of total stress tensor caused by damages, $\omega \in \langle 0,1)$ – dimensionless parameter describing the level of structure damages in material.

The damage parameter $\omega$ introduced in the relation (6) admits value from zero for undamaged material, to one for fully damaged material. It determines a change of unitary bearing surface as a result of microcracks growth and degeneration of material structure. The value of the damage parameter depends in the presented process on the state of stress, hence it must be determined from the kinetic equation

$$\dot{\omega} = f(\sigma), \ \ \omega(t = 0^+) = 0, \quad (7)$$

where: $t$ – time.

The second component $\sigma^D_{ij}$ in the relation (6) expresses an influence of damages on the stress distribution. Taking into account the transformation formula (3) we can get

$$\sigma^D_{ij} = \omega P^+_{ijrs}E_{rskl}\varepsilon^+_{kl} = \omega P^+_{ijrs}E_{rskl}P^+_{klmn}\varepsilon_{mn} = \omega E^+_{ijkl}\varepsilon_{kl}. \quad (8)$$

Finally, the physical equation (6) has the form

$$\sigma_{ij} = E_{ijkl}\varepsilon_{kl} - \omega E^+_{ijkl}\varepsilon_{kl}. \quad (9)$$

Based on (9) in an analogous viscoelastic problem the relation between the stress tensor and the strain tensor can be written as

$$\sigma_{ij} = [E_{ijkl}(t) - \omega E^+_{ijkl}(t)]*d\varepsilon_{kl}, \quad (10)$$

where $E_{ijkl}(t)$ – relaxation functions tensor.

The equation above describes stresses in a quasi linear viscoelastic body with structure damages. So there is need, in real problems, to take into consideration changes of stresses during the whole process of deformations. Therefore, we must use the incremental form of the relation (10)

$$\Delta\sigma_{ij} = [E_{ijkl}(t) - \omega E^+_{ijkl}(t)]*d\Delta\varepsilon_{kl}. \quad (11)$$

It is worth mentioning that the form of the equations (10) and (11), where the linear part is clearly separated from the non-linear one which contains an influence of damages, allows us to formulate stable procedures of computations and global laws (for example reciprocity principle).

## 4. Thermomechanics of the process

The thermomechanical description of damage evolution in material demands us to define independent fields. We will take into considerations the strain tensor, the entropy and the damage parameter $\omega$ as an internal variable. We also assume that viscoelasticity holds and the internal energy is a functional which describes our process

$$\rho U = \rho U(\varepsilon_{ij}, S; \omega), \quad (12)$$

where: $U$ – internal energy, $S$ – entropy, $\rho$ – mass density.

These assumptions allow us to obtain the residual inequality [2, 6]

$$-\rho\dot{U} + \theta\rho\dot{S} + \sigma_{ij}\dot{\varepsilon}_{ij} - \frac{q_i\,\theta_{,i}}{T_0} \geq 0, \quad (13)$$

where: $S$ – entropy, $q_i$ – heat flux, $T_0$ – initial temperature, $\theta$ – temperature gain, $(\dot{\ })$ - local time derivative,

which is obtained from the balance of energy and the inequality of entropy increase, as a result of "typical thermomechanical considerations". It is worth mentioning that the condition (13) should be satisfied in every real process. Then we can approximate the internal energy (12) with the following polynomial

$$\rho U = \rho U_0 - \sigma^0_{ij} * d\varepsilon_{ij} + \frac{1}{2}m(\rho S)^2 + \frac{1}{2}E_{ijkl}(t) * \\ * d\varepsilon_{kl} * d\varepsilon_{ij} - \frac{1}{2}\omega E^+_{ijkl}(t) * d\varepsilon_{kl} * d\varepsilon_{ij} + R, \quad (14)$$

where: $U_0$, $\sigma^0_{ij}$ – initial internal energy and initial stress tensor, $m$ – material constant, $R$ – remainder of polynomial.

The time derivative of internal energy expressed as (14) introduced to the expression (13) will finally give the inequality

$$(\theta - m\rho S)\rho\dot{S} + (\sigma_{ij} - E_{ijkl}(t) * d\varepsilon_{kl} + \omega E^+_{ijkl}(t) * \\ * d\varepsilon_{kl} + \sigma^0_{ij})\dot{\varepsilon}_{ij} - \frac{1}{2}\frac{\partial}{\partial t}(E_{ijkl}(t) * d\varepsilon_{kl} * d\varepsilon_{ij}) + \\ + \frac{1}{2}\frac{\partial}{\partial t}(\omega E^+_{ijkl}(t) * d\varepsilon_{kl} * d\varepsilon_{ij}) + \frac{1}{2}\dot{\omega}E^+_{ijkl}(t) * d\varepsilon_{kl} * \\ * d\varepsilon_{ij} - \frac{q_i\,\theta_{,i}}{T_0} \geq 0. \quad (15)$$

An assumption that the above condition holds in any real state of the strain tensor and the entropy is tantamount to the following relations

$$\theta = m\rho S,$$

$$\sigma_{ij} = E_{ijkl}(t) * d\varepsilon_{kl} - \omega E^+_{ijkl}(t) * d\varepsilon_{kl} - \sigma^0_{ij},$$

$$-\frac{1}{2}\frac{\partial}{\partial t}(E_{ijkl}(t) * d\varepsilon_{kl} * d\varepsilon_{ij}) + \frac{1}{2}\frac{\partial}{\partial t}(\omega E^+_{ijkl}(t) * d\varepsilon_{kl} * \\ * d\varepsilon_{ij}) + \frac{1}{2}\dot{\omega}E^+_{ijkl}(t) * d\varepsilon_{kl} * d\varepsilon_{ij} \geq 0, \quad (16)$$

$$\rho_i\,\theta_{,i} \backslash \geq 0.$$

Neglecting thermal influences, initial stresses and viscous power dissipation

$$\theta \cong 0, q_1 \cong 0, \sigma^0_{ij} \cong 0$$

$$-\frac{1}{2}\frac{\partial}{\partial t}(E_{ijkl}(t) * d\varepsilon_{kl} * d\varepsilon_{ij}) + \frac{1}{2}\frac{\partial}{\partial t}(\omega E^{+}_{ijkl}(t) * d\varepsilon_{kl} * * d\varepsilon_{ij}) \cong 0 \tag{17}$$

we will get the physical equation for the stress tensor

$$\sigma_{ij} = E_{ijkl}(t) * d\varepsilon_{kl} - \omega E^{+}_{ijkl}(t) * d\varepsilon_{kl}, \tag{18}$$

and the condition for the damage evolution

$$\frac{1}{2}\,\dot{\omega} E^{+}_{ijkl}(t) * d\varepsilon_{kl} * d\varepsilon_{ij} \geq 0. \tag{19}$$

In the principal stresses space we can obtain from (19)

$$\frac{1}{2}\,\dot{\omega}\langle\sigma_k\rangle * d\varepsilon_k \geq 0. \tag{20}$$

From the condition above we can conclude that the increase of damage parameter is closely connected with the work of the principal tensile stresses on the strains. Making an assumption that the damage process is an irreversible one we can postulate the damage evolution equation

$$\dot{\omega} = CE^{+}_{ijkl}(t) * d\varepsilon_{kl} * d\varepsilon_{ij},\ C \geq 0, \tag{21}$$

where: $C$ – material parameter.

## 5. Determination of the damage parameter

The damage evolution and connected with this process the strength drop are dependent on the velocity of strains and stresses. The increase of velocity of a load raises the final strength, reduces the strains at the maximal stress and the dependence stress-strain is closer to a linear one. This phenomenon is connected with the inertia of concrete on mikrocracks. Tensile stresses have more considerable influence on the change of strength than compressive ones [4] (Fig. 2). The experimental description of this process is presented in the work [1], where the tensile strength, the strain at the maximal stress for concrete are dependent on the velocity of strain

$$f_{ctm} = f_{ctm,0}\left(\frac{\dot{\varepsilon}}{\dot{\varepsilon}_0}\right)^{\delta_1},\ \ \delta_1 = \frac{1{,}016}{10 + 0{,}5\, f_{ctm,0}}, \tag{22}$$

$$\varepsilon_{ct} = \varepsilon_{ct,0}\left(\frac{\dot{\varepsilon}}{\dot{\varepsilon}_0}\right)^{\delta_2},\ \ \delta_2 = -0{,}020, \tag{23}$$

where: $\dot{\varepsilon}_0$ – reference velocity of strain (in the work [1] $\dot{\varepsilon}_0 = 3 \cdot 10^{-6}$ [$s^{-1}$]), $\dot{\varepsilon}$ – velocity of strain, $f_{ctm,0}$ – tensile strength for reference velocity of strain, $\varepsilon_{ct,0}$ – strain at tensile strength for reference velocity of strain, $\varepsilon_{ct}$ – strain at tensile strength.

The determination of the value of damage parameter from the relation (21) can be simplified after taking into account the assumptions presented in the introduction. Damages in material appear immediately after imposing a load, in contrast to the process of creep, so the damage evolution equation can be written as

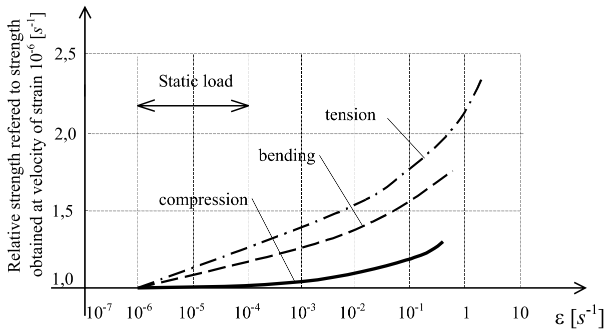


*Fig. 2. Influence of velocity of strain on relative strength of concrete during tension, bending and compression.*

$$\dot{\omega} = CE^{+}_{ijkl}\,\varepsilon_{kl}\,\varepsilon_{ij},\ C \geq 0. \tag{24}$$

Hence, in the case of increase of strain at a constant velocity under uniaxial state of stress, the damage parameter is described by the following formula

$$\omega = \frac{CE_0\varepsilon^3}{3\dot{\varepsilon}},\ \dot{\varepsilon} = const., \tag{25}$$

where: $E_0$ – initial Young`s modulus $\varepsilon$ – strain in uniaxial state of stress.

A typical curve determining the relation $\sigma - \varepsilon$ in uniaxial state of tensile stress for concrete is illustrated in Fig. 3,

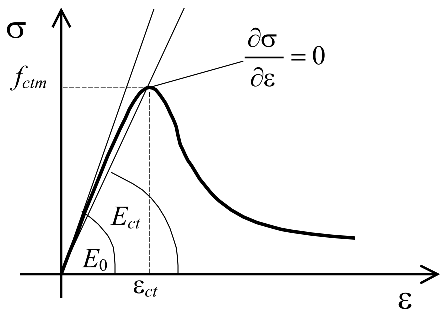


*Fig. 3. Relation in uniaxial state of stress.*

where: $E_{ct}$ – secant Young`s modulus, $f_{ctm}$ – tensile strength.

Taking into account that the tangent to the curve $\sigma - \varepsilon$ in point ($\varepsilon_{ct}, f_{ctm}$) is equal to zero we can calculate the unknown material parameter $C$

$$C = \frac{9\dot{\varepsilon}E^2_{ct}}{16 f^3_{ctm}}, \tag{26}$$

which is a function of velocity of strain. That means we must use the incremental form of relation (25) to describe the damage parameter for changeable velocity of strain

$$\Delta\omega = \frac{C(\dot{\varepsilon})E_0\,\Delta\varepsilon^3}{3\dot{\varepsilon}}. \tag{27}$$

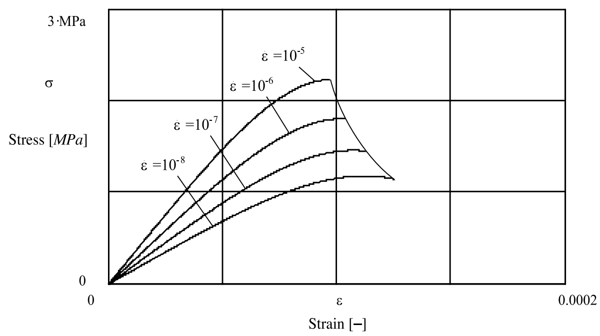


*Fig. 4. Stress at tension for different velocities of strain.*

Character of the physical relation (9) in the uniaxial state of stress with taking into account relations (25) and (26) is shown in Fig. 4. (for example, parameters of concrete $\dot{\varepsilon}_0 = 3 \cdot 10^{-6} [s^{-1}]$, $f_{ctm,0} = 2$ [MPa], $\varepsilon_{ct.0} = 10^{-4}$ [–]).

## 6. Conclusions

1. The physical equations including the influence of principal tensile stresses on destruction of concrete are formulated.
2. The limitation for the process of damage evolution is obtained from the thermomechanical considerations.
3. Presented formulas describing the damage evolution can be helpful for the estimation of exploitation time of concrete.

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