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# POZDĹŽNY ŠMYKOVÝ TOK V PODPEROVEJ OBLASTI TRÁMU S PREVISLÝM KONCOM

## LONGITUDINAL SHEAR FLOW AT THE SUPPORT AREA OF BEAM WITH AN OVERHANGING CANTILEVER

*Príspevok sa zaoberá šmykovým tokom v tráme pri jeho podpere. Skúma najmä konzolovú časť, ktorú v skutočnosti majú všetky typy trém bez ohľadu na materiál a rozpätie. Parametrická štúdia rozdelenia šmykových napätí po výške prierezu sa realizovala metódou konečných prvkov. Uvádza výsledky najmä pre konzolovú časť. Vyúsťuje do formulácie záverov pre prax.*

*The paper deals with longitudinal shear flow of beams near the support area. Special interest is focused on the overhanging part, which exists in all kinds of beams irrespective of material and span. A parametric study on shear stress distribution at several horizontal height levels by using FEM is presented. Special emphasis is paid to the cantilever length. Finally, conclusions for practical applications are summarized.*

### 1. Introduction

In practical design of bridge girders, knowledge of shear stress distribution at a certain horizontal height level of a beam is often needed. As example, the design of shear connectors of the commonly used steel-concrete, wood-concrete or concrete-concrete composite girders can be mentioned. This knowledge is also required when designing fillet welds of welded steel plate girders.

For simple calculations, the beam theory is obviously used. Thus, when the girder width is not taken into account, the shear flow is represented by the well-known formula

$$n_r(x, z) = \frac{V(x) \cdot S'(z)}{I}, \quad (1)$$

where  $V(x)$  is the internal shear force,  $S'(z)$  is the static moment of partial area with respect to the vertical level considered and  $I$  represents moment of inertia, respectively.

However, in some cases this theory produces wrong prediction of stress distribution. This can be clearly seen above support areas because of two main reasons. Firstly, the beam is not a line but has indispensable height and is supported and loaded not on its axis but at the horizontal and bottom surfaces, respectively. Secondly, the support reaction forces do not act at the corner points of the girder, which means that additional cantilever parts always exist. Thus, the longitudinal shear stresses are affected by local stresses caused by reaction, which can be considered as local force. The disc (wall) effect in this area is usually significant and

therefore this area should be considered as a two-dimensional problem at least.

### 2. Theoretical background

Stresses at the support area can be described by the disc (plane) theory according to the basic equation of theoretical mechanics, namely

$$\nabla^4 \phi(x, z) = 0, \quad (2)$$

where  $\phi(x, z)$  is Airy's stress function. The solution of this problem is complicated. An improvement of the beam theory, based on several simplifications and solutions of partial differential equation (2), can be found. When the reaction force is placed at the corner point, the solution consists of superposition of stresses caused by external load, reaction force and stresses due to boundary conditions. No detailed comments are presented hereby since such procedure can be found in mathematical form e.g. in [1]. A similar method based on several steps may be developed for the case of "infinite" long cantilever behind the support. Anyway, in practical cases the cantilever has a finite length. In such conditions, it is very difficult to find a practically applicable analytical solution and that is why a numerical approach based on FEM seems to be needed. Although no analytical rules for shear stress distribution can be obtained from FEM, it is important to realize, that this approach offers a possibility to investigate the influence of various parameters. One of those parametric studies is presented here.

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### 3. Parametric study

In this study, the effect of the size of a cantilever located beyond the support line is discussed. A one-meter high and 10 meters span simply supported beam with rectangular cross-section is considered (Fig. 1).

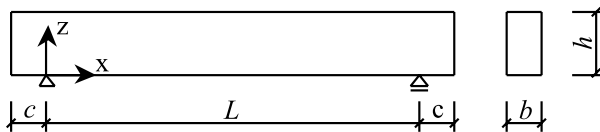


Fig. 1. Scheme of the beam.  $L = 10$  m,  $h = 1$  m,  $b$  equals to unit width and  $c$  is variable.

Because all stresses are supposed to depend linearly on the width, this dimension can be taken equal to unity and thereafter its further consideration is not needed. The cantilever length,  $c$ , is

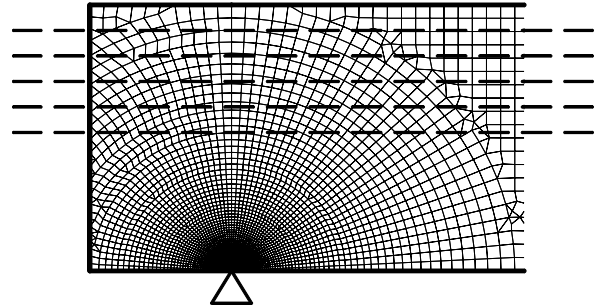


Fig. 2. Part of FEM model with studied horizontal levels

For example, in Fig. 3, results obtained by using six different cantilever lengths with  $S'/I$  ratio equal to 1.44 are presented. Only the cantilever part and a length corresponding to one fifth of the span are shown. In Fig. 4 is shown the same relationship but in the case of  $S'/I$  ratio equal to 0.96.

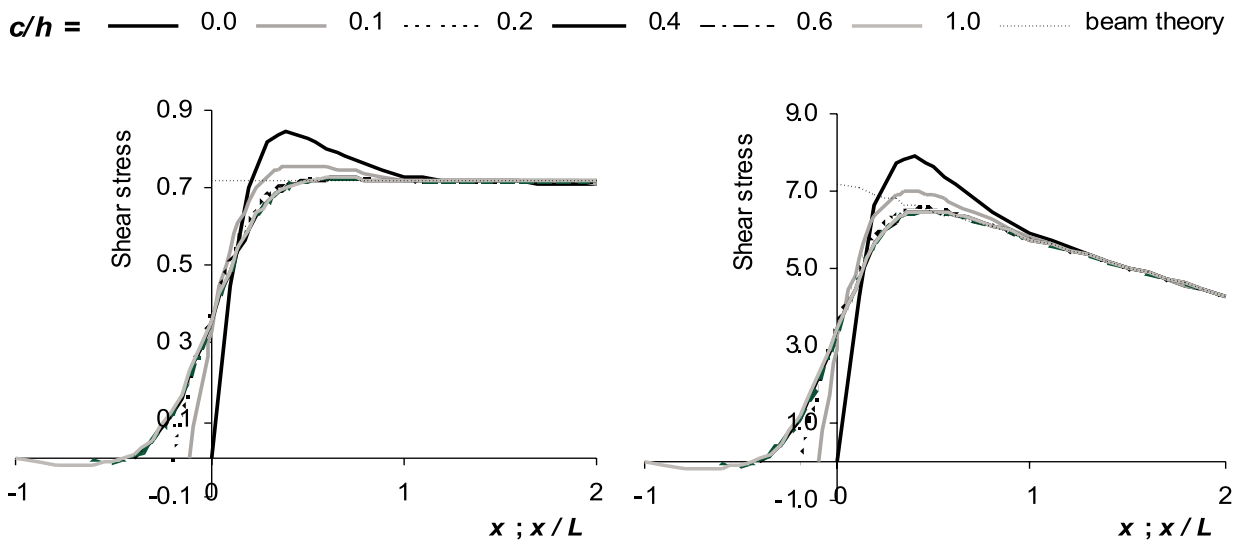


Fig. 3. Longitudinal shear stresses at support area, when  $S'/I = 1.44$  and the load consists of a) a single force in the middle and b) uniformly distributed load over the span.

varied from zero to the height of the beam, i.e. from  $0 \cdot h$  to  $1.0 \cdot h$ . Two load cases are considered. The first case consists of one unit force in the middle of the span. The uniformly distributed unit load between the supports is considered in the other load case. The longitudinal shear flow is investigated at the horizontal levels ("z" coordinate according to Fig. 1) with  $z/h$  ratio equal to 0.5, 0.6, 0.7, 0.8 and 0.9, respectively. These positions correspond to the  $S'/I$  ratios 1.5, 1.44, 1.26, 0.96 and 0.54, respectively. The ratio  $S'/I$  reflects the value required in equation (1).

A combined mesh of quadrangle and triangle finite elements in FEM program was chosen. The support was considered as a point constraint. In reality, certain length over which is reaction distributed always exist. However, when very fine mesh of FE is used (Fig. 2) and only further horizontal levels are studied, the point reaction can be used with sufficient accuracy as well.

In Fig. 5, a detailed view on the cantilever part is presented when  $S'/I$  ratio is equal to 0.96. From the graphs and other results not presented here, it is evident that a shear stress peak can occur in front of the support within an area whose length is approximately equal to the height of the beam. The length of cantilever has influence on distribution of this peak behind the support, however, the overhang longer than  $0.4 \cdot h$  is found to be useless.

From the next picture, Fig. 6, it is also obvious that in the support cross-section, around 50 % of the shear flow can remain at each considered horizontal level, if the cantilever is longer than  $0.2 \cdot h$ .

The percentage increase of longitudinal shear flow in the peaks mentioned before is expressed in Fig. 7. It is visible that when the cantilever length is more than  $0.3 \cdot h \sim 0.4 \cdot h$ , the stress peak is less

$c/h =$  — 0.0 — 0.1 — 0.2 — 0.4 — 0.6 — 1.0 — beam theory

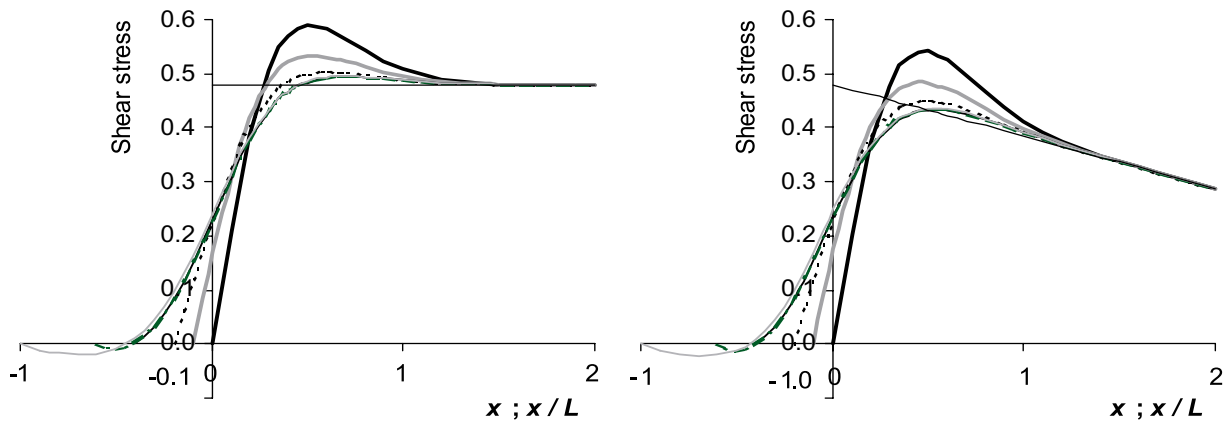


Fig. 4. Longitudinal shear stresses at support area, when  $S'/I = 0.96$  and the load consists of a) a single force in the middle and b) uniformly distributed load over the span.

than 5 % at all considered levels and probably can be neglected. When the cantilever becomes shorter, local stresses can cause a significant stress peak compared to that obtained through the beam theory and especially at the higher horizontal levels of the girder.

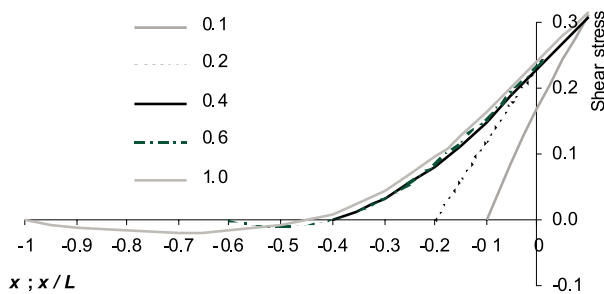


Fig. 5. Longitudinal shear stresses at cantilever area for  $S'/I$  ratio equal to 0.96 when a single force is situated in the middle of the span

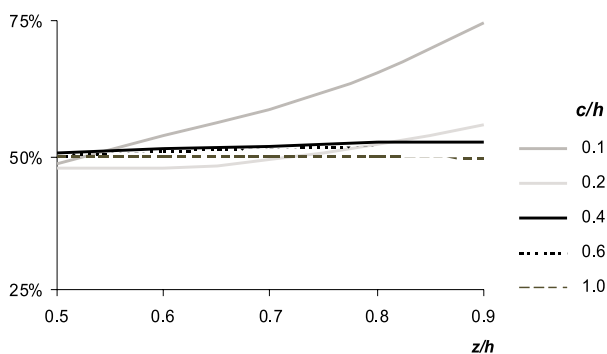


Fig. 6. Percentage remains of longitudinal shear stresses in the support cross-section compared to the beam theory.

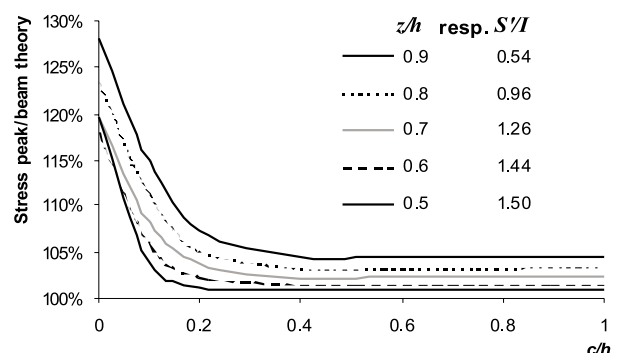


Fig. 7. Percentage increase of longitudinal shear flow in the stress peak compared to the result obtained by the beam theory.

#### 4 Conclusions

The main aim of the article was not to solve the problem theoretically, but to show the impact of cantilever length on horizontal shear flow at support area.

All of the results presented above refer to the beam with span of 10 m. In the parametric study also beam length of 15, 20 and 30 m were considered, therefore some general summary could be formulated.

As conclusions for composite girders it can be stated that, in the case of non-ductile shear connectors, the most affected connectors are those located about  $0.5 \cdot h$  from the support. This conclusion suits quite well with the conclusions presented in [2] and [3]. The shear force, which has to be carried, is higher than the one computed according to the beam theory. The increase of this shear flow is dependant on the length of cantilever and  $S'/I$  ratio mentioned before (Fig. 7). The cantilever length of  $0.3 \cdot h$  seems to be the most suitable one, because no additional lengthening leads to greater reduction of the shear peak. Therefore, shear connectors placed in the cantilever further away than  $0.3 \cdot h \sim 0.4 \cdot h$  from the

support line seem to be ineffective. In addition, when the overhang is longer than this value, the stress peak discussed above can be neglected as well. Consequently, in the case of non-ductile connectors, the longitudinal shear force can be considered according to Fig. 8.

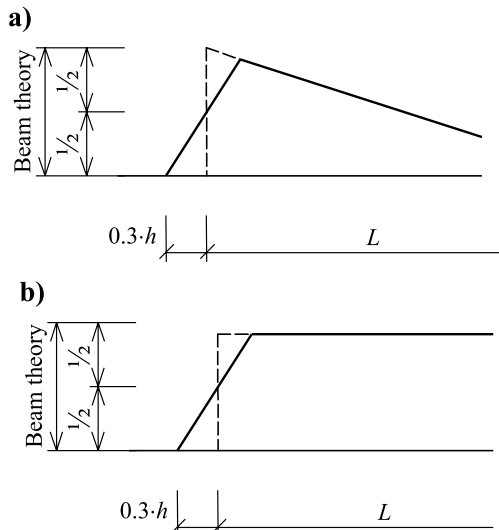


Fig. 8. Simplified shear flow distribution in support region for a beam loaded by a) the uniformly distributed load between supports and b) a force in the middle of the span

Therefore, for girders with cantilevers longer than  $0.3 \cdot h$ , the approach based on beam theory should be modified as follows:

- shear flow continues behind the support line and linearly decreases towards zero at a distance of  $0.3 \cdot h \sim 0.35 \cdot h$ ;
- in the support cross-section, 50 % of the shear computed according to the simple beam theory can be considered;
- in the case of a long overhang, the shear peak can be ignored.

This simplified distribution shown in Fig. 8 can be used when determining the number and location of shear connectors in composite girders assuming that the length of overhanging cantilever is big enough. In the case when uniformly distributed load is placed not only between supports but also on cantilevers, similar distribution as in Fig. 8-a) can be adopted. If the cantilever length is not much different from the value  $0.3 \cdot h$ , this simplification is sufficiently accurate.

Finally, it has to be noticed that the conclusions presented above may not be valid when plastic or deformable behaviour of shear connectors is considered or expected. In this case, the shear flow is better redistributed along the girder length and behind the support, which means that a longer cantilever can be effective as well. In addition, all shear flows or connector forces discussed above were calculated without considering the effect of shrinkage in the concrete part of a composite cross-section, which means that significant forces can escape attention.

## References

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- [2] SZABÓ, B.: *Shear Force Distribution of Headed Studs of Composite Simply Supported Beams Subjected to Temperature Difference*. Proceedings of Conference "Eurosteel 99", Praha 1999. Pp. 557 - 560.
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