

# THE STRESS-STRAIN STATE OF THE TUNNEL LINING THAT CROSSES THE FAULT ZONE OF SOIL BLOCKS DURING AN EARTHQUAKE

Mark R. Miller<sup>1</sup>, Evgeniy Y. Titov<sup>2</sup>, Sergey S. Kharitonov<sup>2\*</sup>, Yong Fang<sup>1</sup>

<sup>1</sup>Department of Civil Engineering, Southwest Jiaotong University, Chengdu, Sichuan, China

<sup>2</sup>Department of Bridges and Tunnels, Russian University of Transport (MIIT), Moscow, Russia

\*E-mail of corresponding author: mr.kharitonov.94@mail.ru

## Resume

The study examines the question of the tunnel behavior under seismic or geophysical load in the zone of changes in the hardness of the surrounding soil mass. In the course of the study, the internal forces and displacements arising in the structure of a tunnel in the zone of intersection of the boundaries of soil layers with different properties, in the case when these layers move relative to each other, were determined by analytical and numerical solutions. The data obtained by the analytical method was compared to numerical models using practical examples.

## Article info

Received 17 January 2021

Accepted 31 March 2021

Online 22 October 2021

## Keywords:

transport tunnels,  
beam on an elastic foundation,  
finite elements,  
tunnel damage,  
earthquake

Available online: <https://doi.org/10.26552/com.C.2022.1.D9-D22>

ISSN 1335-4205 (print version)

ISSN 2585-7878 (online version)

## 1 Introduction

During an earthquake, soil layers with different properties often move in different ways. Tunnels that cross such borders may be damaged [1-2]. Tunnels located in soft ground can be considered as beams in an elastic medium (or considered as beams on an elastic foundation), Figure 1. As a rule, faults are usually the boundaries of soil layers with different engineering and geological characteristics. This study examines the stress-strain state of the tunnel lining that crosses the fault zone of two soil blocks [3]. At this stage of calculation, the goal is to obtain analytical and numerical solutions and then compare them. The initial data are parameters of the lining cross-section, characteristics of the soil and value of the relative displacement of the soil layers [5].

## 2 The fault is perpendicular to the tunnel axis

### 2.1 Analytical method

Differential equation of the bending beam, (Figure 2) [7-8], is:

$$EI_x \frac{d^4 y}{dx^4} + kby = q. \quad (1)$$

The well-known solution of Equation (1) has the form:

$$y = e^{-\beta x} (C_1 \cos \beta x + C_2 \sin \beta x) + e^{\beta x} (C_3 \cos \beta x + C_4 \sin \beta x) + v_0(q), \quad (2)$$

where  $\beta = \sqrt{\frac{kb}{4EI_x}}$ , here  $k$  is a coefficient of the subgrade reaction,  $b$  is the width of a beam,  $E$  is Young modulus,  $I_x$  is a moment of inertia.

In this case, the deformed view is as in Figure 3.

For simplifying the solution, the model of the semi-infinite beam on an elastic foundation is used (Figure 4), when  $0 \leq x \leq \infty$ . In this case, if  $x \rightarrow \infty, e^x \rightarrow \infty$ , there is no physical meaning, so  $C_3 - C_4 = 0$ . In addition, from initial data follows that  $q = 0$ .

In view of the above, Equation (2) will take a form:

$$y = e^{-\beta x} (C_1 \cos \beta x + C_2 \sin \beta x). \quad (3)$$

To define the integration constants  $C_1$  and  $C_2$ , one has to twice take the derivative of a function in Equation (3). Below is the sequence of actions for this operation.

$$\begin{aligned} y &= C_1 e^{-\beta x} \cos \beta x + C_2 e^{-\beta x} \sin \beta x \\ y' &= C_1 (e^{-\beta x} \cos \beta x)' + C_2 (e^{-\beta x} \sin \beta x)' \\ 1. (e^{-\beta x} \cos \beta x)' &= -\beta e^{-\beta x} \cos \beta x + \\ &+ \beta e^{-\beta x} (-\sin \beta x) = \beta e^{-\beta x} (-\cos \beta x - \sin \beta x) \end{aligned} \quad (4)$$

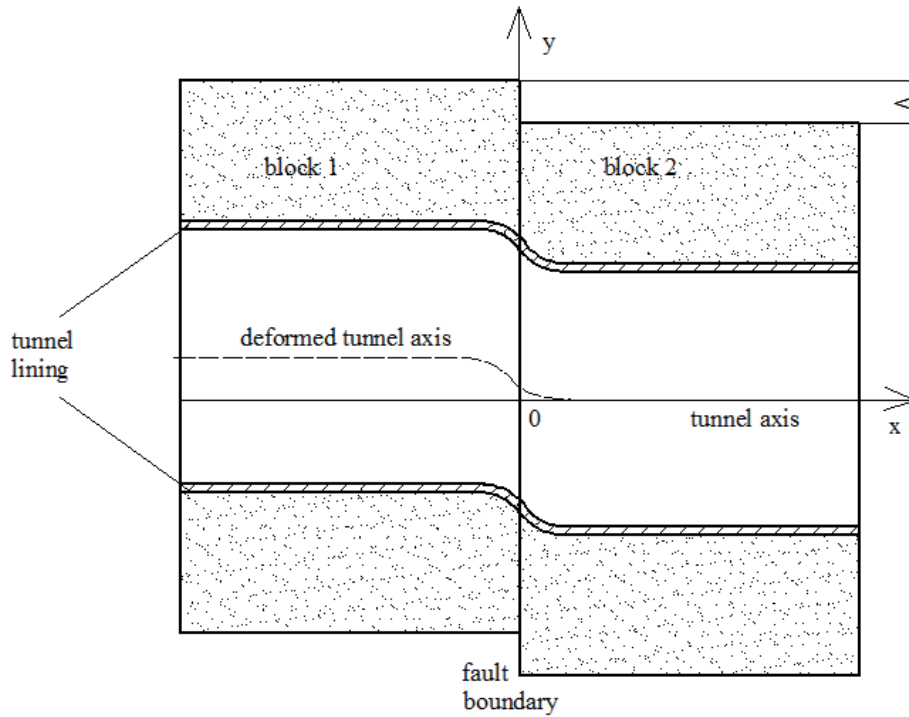


Figure 1 Scheme of a tunnel through fault fracture zone

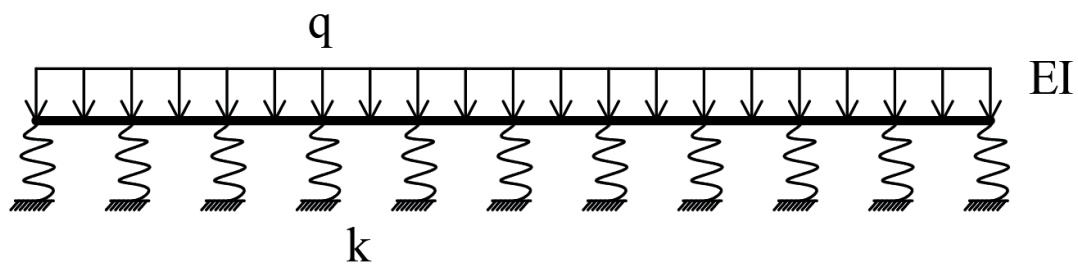


Figure 2 Classical beam on elastic foundation using the Winkler assumption

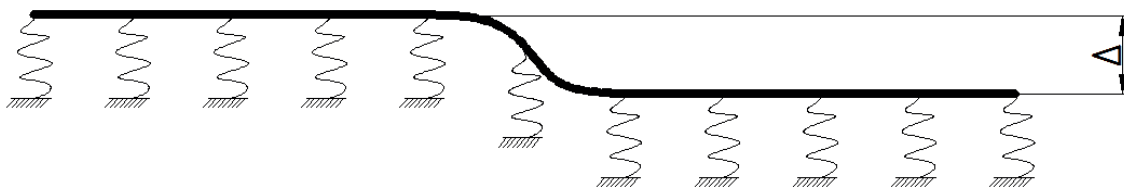


Figure 3 Deformed view of the calculation scheme

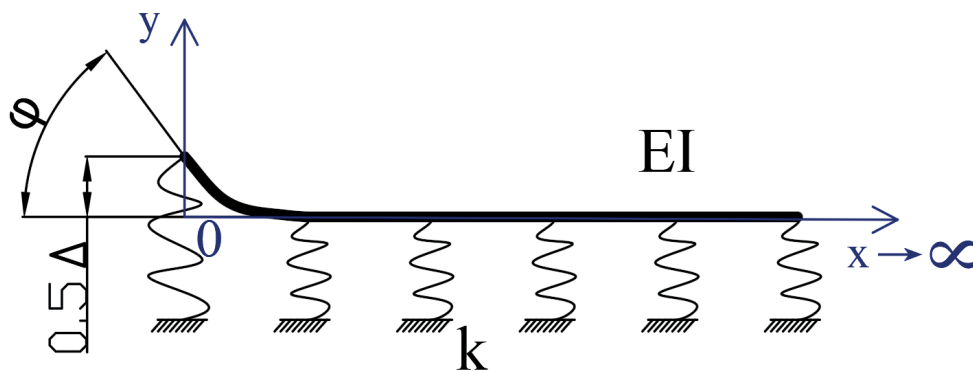


Figure 4 Accepted calculation scheme

$$\begin{aligned}
 2. (e^{-\beta x} \sin \beta x)' &= -\beta e^{-\beta x} \sin \beta x + \\
 &+ \beta e^{-\beta x} \cos \beta x = \beta e^{-\beta x} (-\sin \beta x + \cos \beta x) \\
 y' &= C_1 [\beta e^{-\beta x} (-\cos \beta x - \sin \beta x)] + \\
 &+ C_2 [\beta e^{-\beta x} (-\sin \beta x + \cos \beta x)].
 \end{aligned}
 \tag{4}$$

$$\begin{aligned}
 y'' &= C_1 \beta [-e^{-\beta x} \cos \beta x - e^{-\beta x} \sin \beta x]' + \\
 &+ C_2 \beta [e^{-\beta x} \cos \beta x - e^{-\beta x} \sin \beta x]' \\
 y'' &= C_1 \beta \left[ -\beta e^{-\beta x} (-\cos \beta x - \sin \beta x) - \right. \\
 &\left. -\beta e^{-\beta x} (-\sin \beta x + \cos \beta x) \right] + \\
 &+ C_2 \beta \left[ \beta e^{-\beta x} (-\cos \beta x - \sin \beta x) - \right. \\
 &\left. -\beta e^{-\beta x} (-\sin \beta x + \cos \beta x) \right] \\
 y'' &= C_1 \beta^2 e^{-\beta x} \left[ (\cos \beta x + \sin \beta x) - \right. \\
 &\left. -(\sin \beta x + \cos \beta x) \right] + \\
 &+ C_2 \beta^2 e^{-\beta x} \left[ (-\cos \beta x - \sin \beta x) - \right. \\
 &\left. -(-\sin \beta x + \cos \beta x) \right] \\
 y'' &= C_1 \beta^2 e^{-\beta x} \left[ \cos \beta x + \sin \beta x + \right. \\
 &\left. + \sin \beta x - \cos \beta x \right] + \\
 &+ C_2 \beta^2 e^{-\beta x} \left[ -\cos \beta x + \sin \beta x + \right. \\
 &\left. + \sin \beta x - \cos \beta x \right] \\
 y'' &= C_1 \beta^2 e^{-\beta x} * 2 \sin \beta x - C_2 \beta^2 e^{-\beta x} * 2 \cos \beta x \\
 y'' &= 2\beta^2 e^{-\beta x} (C_1 \sin \beta x - C_2 \cos \beta x).
 \end{aligned}
 \tag{5}$$

If the properties of the neighboring ground-blocks and bending stiffness of the tunnel lining are constant, for right-hand part of an infinite beam the following boundary conditions can be used [9]:

$$\left\{ \begin{aligned}
 y(0) &= \frac{\Delta}{2} \\
 \frac{d^2 y(0)}{dx^2} &= 0 \Rightarrow y''(0) = 0
 \end{aligned} \right.
 \tag{6}$$

When  $x = 0$ , the tangent angle takes the extreme value, the second derivative should be equal to zero, whence follows  $y''(0) = 0 > M(0) = 0$ .

Substituting the boundary conditions in Equations (6), (3) and (5) gives:

$$\begin{aligned}
 1. y''(0) &= \frac{\Delta}{2}: \frac{\Delta}{2} = e^{-\beta \cdot 0} (C_1 \cos \beta * 0 + \\
 &+ C_2 \sin \beta * 0) = \\
 \Rightarrow C_1 &= \frac{\Delta}{2}, \\
 2. y''(0) &= 0: 0 = 2\beta^2 e^{-\beta \cdot 0} (C_1 \sin \beta * 0 - \\
 &- C_2 \cos \beta * 0) = \\
 \Rightarrow C_2 &= 0.
 \end{aligned}
 \tag{7}$$

Taking into the constants found, Equation (3) takes the form:

$$y = \frac{\Delta}{2} e^{-\beta x} \cos \beta x.
 \tag{8}$$

Using the well-known dependencies between the internal force factors (bending moment M and shear force Q) and a deflection function, one can get equations:

$$\begin{aligned}
 \frac{M}{EI} &= -y'' = -2\beta^2 e^{-\beta x} \frac{\Delta}{2} \sin \beta x; \\
 M &= -EI \Delta \beta^2 e^{-\beta x} \sin \beta x,
 \end{aligned}
 \tag{9}$$

$$\begin{aligned}
 \frac{Q}{EI} &= -y''' = -\Delta \beta^3 (e^{-\beta x} \sin \beta x)' = \\
 &= -\Delta \beta^3 [\beta e^{-\beta x} (-\sin \beta x + \cos \beta x)]; \\
 Q &= EI \Delta \beta^3 e^{-\beta x} (\cos \beta x - \sin \beta x).
 \end{aligned}
 \tag{10}$$

The presence of  $e^{-\beta x}$  multiplier in equations indicates that all these functions decrease with increasing distance from the block border ( $x \rightarrow \infty, e^{-\beta x} \rightarrow 0$ ). One

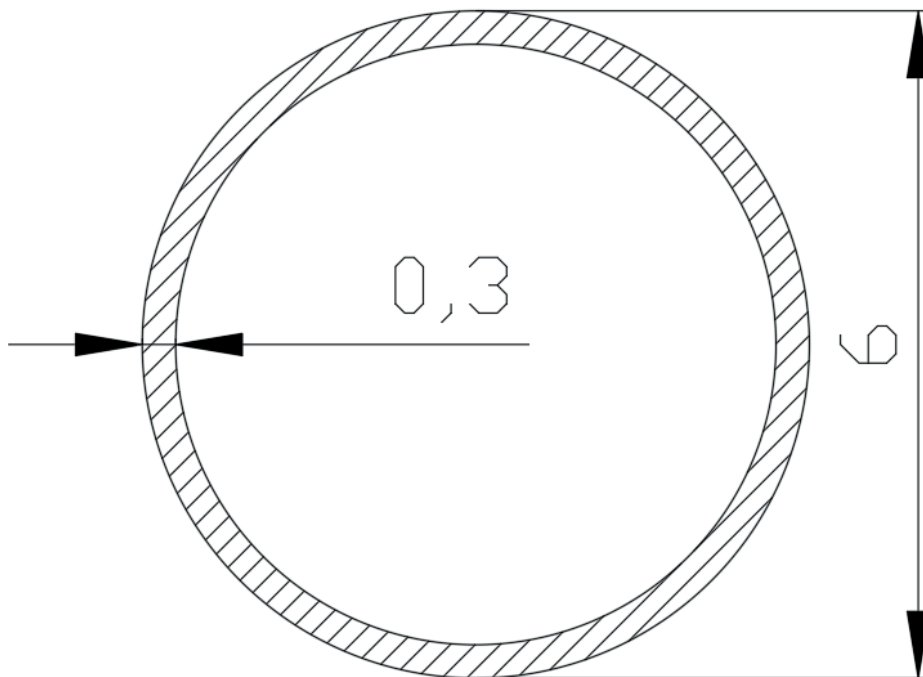


Figure 5 Cross-section area of the tunnel lining



**Figure 6** Part of the finite-element model of a beam on elastic foundation

can assess the zone of influence by function  $e^{-\beta x}$ . If  $x = 0, e^{-\beta \cdot 0} = 1$ .

If  $\beta x = \pi, e^{-\pi} = 0.043$  and with an accuracy of 5% one can say that it equals to zero. In this case, from  $\beta l_{\text{inf}} = \pi$  follows:

$$l_{\text{inf}} = \frac{\pi}{\beta}. \quad (11)$$

## 2.2 Example

As an example, the stress-strain state of the tunnel lining (Figure 5) is considered for the beam element. Parameters of concrete are: Young modulus  $E = 35500 \text{ Mpa}$ , Poisson ratio  $\nu = 0.2$ .

The vertical deflection of left-hand part is  $\Delta = 0.01 \text{ m}$ . The results were obtained by Excel and presented below (comparing the analytical solution to the numerical simulation's one).

## 2.3 Numerical simulation

### 2.3.1 Model of a Winkler's beam on elastic foundation

An analytical solution, using a mathematical model of a beam on elastic foundation, allows to quickly and easily assess the inertial forces factors in the tunnel lining from the displacement of blocks along the fault boundary; however that is not a universal solution. Numerical modeling helps to solve this problem. First, the numerical finite element model was created in the MIDAS GTS NX software, based on the calculation scheme of a Winkler beam on an elastic foundation (Figure 2). The beam nodes are connected to the ground by elastic springs with a step of 1m, the spring stiffness corresponds to the stiffness of the ground foundation. The length of the simulated tunnel section is 200 meters. The external impact is set as a 0.01m upward shift of the left block. The model is shown in Figure 6.

### 2.3.2 Model with the 2D plane strain elements.

The next step in verifying the proposed calculation method is to create a 2D model.

The tunnel lining is modeled by beam finite elements, a concrete is considered as the isotropic elastic material, using parameters like for a model of beam on an elastic foundation. The area of analysis is 200 x 40m. The surrounding ground mass is modeled by the 2D plane strain elements, using Mohr-Coulomb model with parameters: silty clay, Young modulus  $E = 3700 \text{ kPa}$ , Poisson ratio  $\nu = 0.3$ , friction angle  $\varphi = 18.4^\circ$ , cohesion  $c = 33.8 \text{ kPa}$ . A friction was modeled like an interface elements with strength reduction factor  $R_c = 0.5$ . This model is shown in Figure 7.

For the integrity of the analysis, the model of a beam on an elastic foundation was used, as well. In this case, one needs to use the coefficient of a subgrade reaction  $k$ .

Based on an elastic theory, Scott [6] derived the relation between the coefficient of subgrade reaction and a Young modulus, as follow:

$$k = \frac{E}{d(1 - \nu^2)}, \quad (12)$$

where  $d$  is a diameter of a pile.

For the presented method, it is acceptable to take  $d$  as the tunnel diameter. In this case, for the silty clay the coefficient is  $k = 6777 \text{ kN/m}^2$ .

For the convenience of estimating the stress-strain state in each model, the graphs for each case are presented in Figure 8.

## 2.4 Practical application

### 2.4.1 The 1D scheme

Using the numerical simulation model, a study of the changing inertial forces factors was carried with different stiffness of mountain blocks (Figure 9). Getting analytical results is a difficult procedure in solving

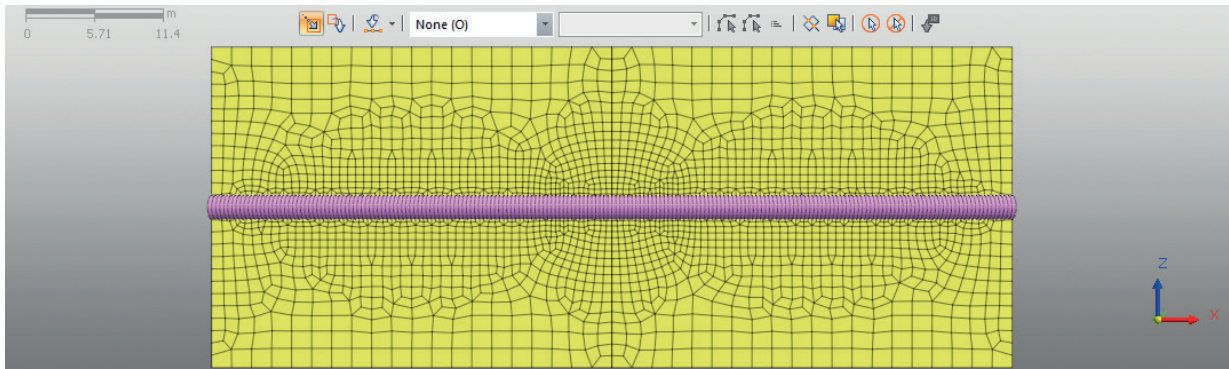
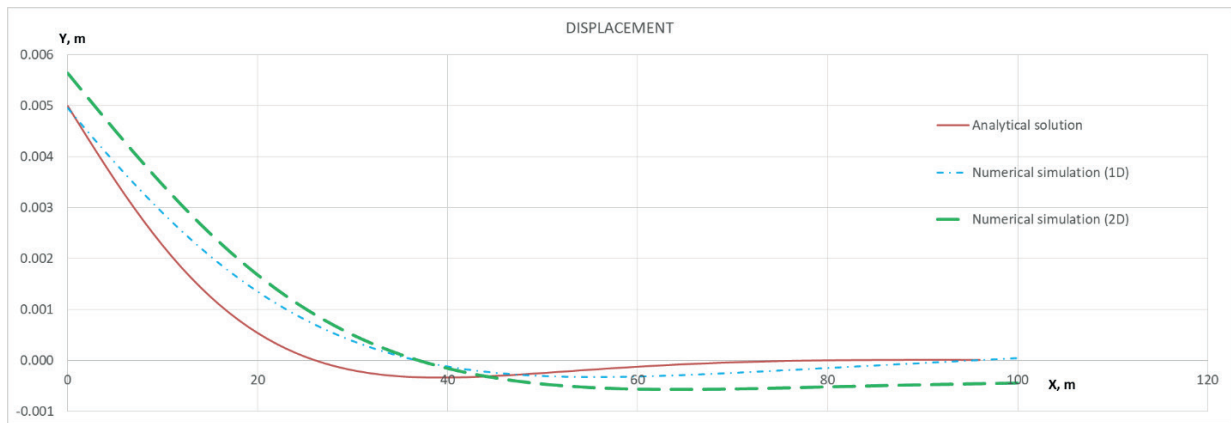
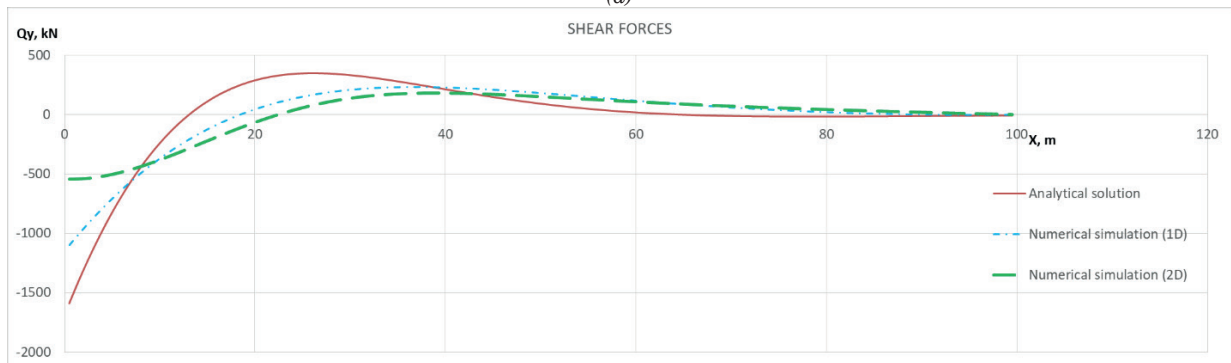


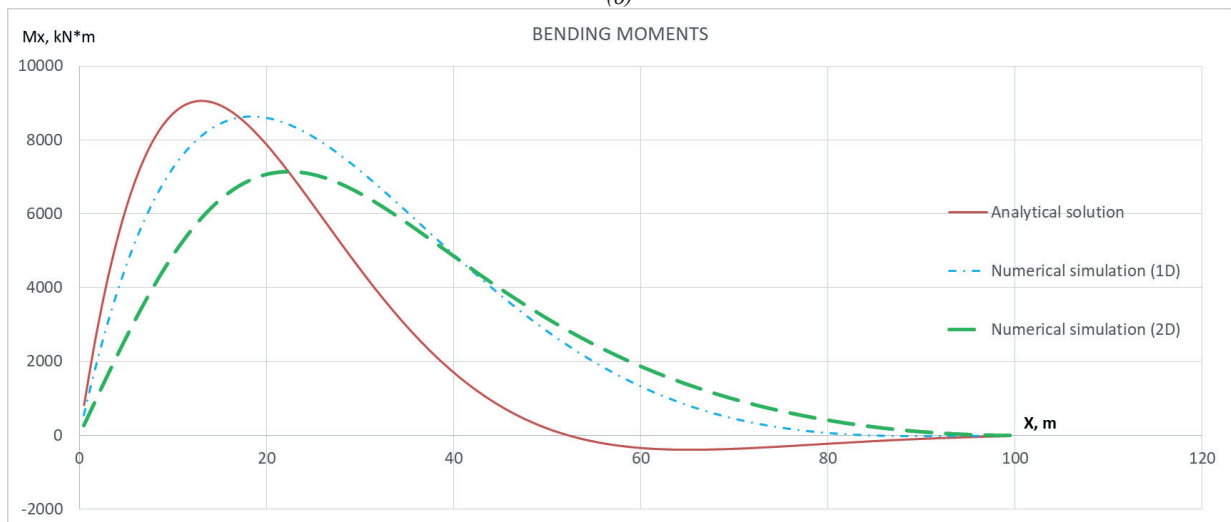
Figure 7 Model with the 2D plane strain elements



(a)

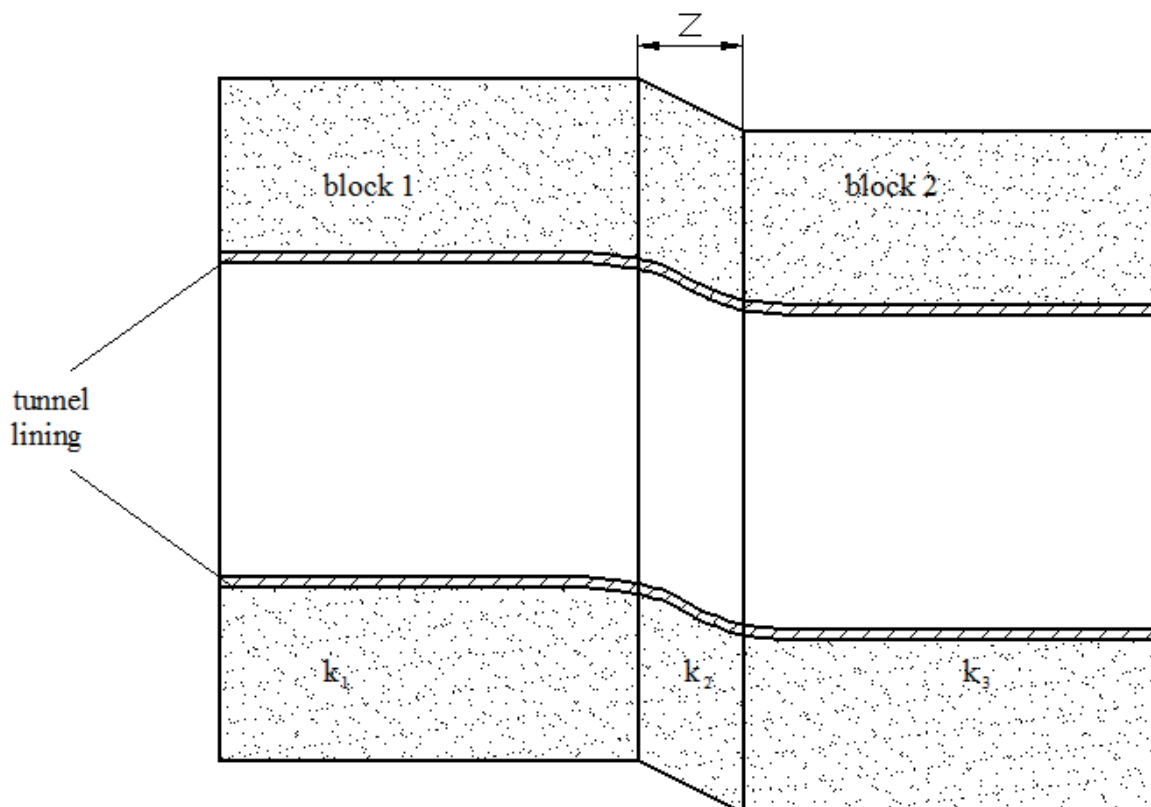


(b)



(c)

Figure 8 Results for the silty clay subgrade. Diagram of displacement (a), shear forces (b) and the bending moments (c)



**Figure 9** Using different coefficients subgrade reaction

such problems, because due to different stiffness of the rock blocks, the deformation and stress plots are not symmetrical.

The stiffness of the first block is characterized by a coefficient of subgrade reaction  $k_1$ , the stiffness of the second block by  $k_3$  and the filling space “z” between blocks by  $k_2$  [9-10].

- a) An effect of the distance  $z$  on internal forces in the tunnel lining

Consider the following additional initial data:

$$k_1 = 40000 \text{ kN/m}^2, k_2 = 0.2k_1 = 8000 \text{ kN/m}^2, \\ k_3 = k_1 = 40000 \text{ kN/m}^2$$

The distance  $z$  varies from 10 to 100 meters. The results are presented in Figure 10.

- b) An effect of the value  $k_2$  on internal forces in the tunnel lining

In this case consider the following additional initial data:

$$k_1 = 8000 \text{ kN/m}^2, k_2 = 10k_1 = 80000 \text{ kN/m}^2, z = 40 \text{ m.}$$

The coefficient  $k_2$  takes values from 8000 to 80000  $\text{kN/m}^2$ . The results are presented in Figure 11. The most interesting is that with increase in the subgrade reaction coefficient, internal force factors increase too and the displacement diagram has t changed slightly.

#### 2.4.2 The 2D scheme

Using this approach, similar operations were performed with the 2D scheme. For connection of the soil

models of the 1D and 2D schemes, the Scott Equation (12) is used.

- a) An effect of the distance  $z$  on internal forces in the tunnel lining

Consider the following additional initial data:

$$E_1 = 218400 \text{ kN/m}^2, E_2 = 0.2E_1 = 43680 \text{ kN/m}^2, \\ E_3 = E_1 = 218400 \text{ kN/m}^2$$

The distance  $z$  varies from 10 to 100 meters. The results are presented in Figure 12.

- b) An effect of the value  $k_2$  on internal forces in the tunnel lining

In this case consider the following additional initial data:

$$E_1 = 43680 \text{ kN/m}^2, E_3 = 10E_1 = 436800 \text{ kN/m}^2, z = 40 \text{ m.}$$

$E_2$  takes values from 43680 to 436800  $\text{kN/m}^2$ . The results are presented in Figure 13.

### 3 The fault is along to the tunnel axis

#### 3.1 Analytical method

Consider a tunnel with length  $2L$ , external diameter  $d$ , and with compressive (or tensile) stiffness  $EA$ . Using the Winkler model to describe the interaction of the surrounding soil mass and the tunnel structure, the coefficient of elastic resistance at the shift  $k_s$  is introduced (Figure 14).

The movement of the tunnel cross-sections  $w$  along the  $x$  axis is described by the following differential

equation [5]:

$$EA \frac{d^2 w}{dx^2} - \pi d k_s w = 0. \tag{13}$$

### 3.2 Example

For example, a tunnel lining with the above

characteristics was adopted:  $A = 5.372 \text{ m}^2$ ,  $d = 6 \text{ m}$ . Coefficient  $k_s$  is adopted as a quarter of the normal coefficient of subgrade reaction. For silty clay:  $k_s = 2000 \text{ kN/m}^2$ .

For the ease of calculation, consider the right-hand cut-off part in accordance with the methods of strength of materials (Figure 15).

The well-known differential relationship between

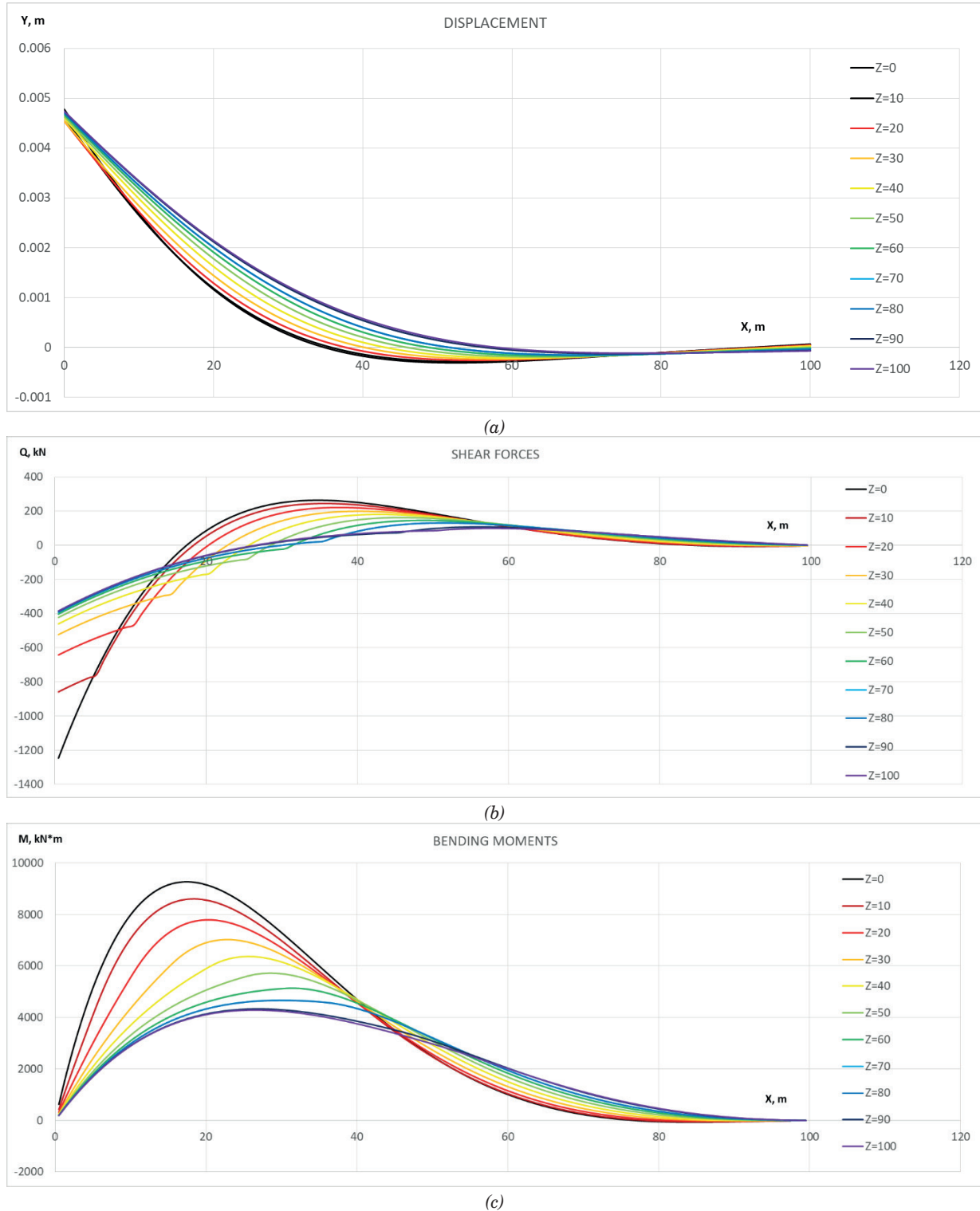


Figure 10 Diagram of displacement (a), shear forces(b) and bending moments(c) in terms of  $z$  (1D scheme)

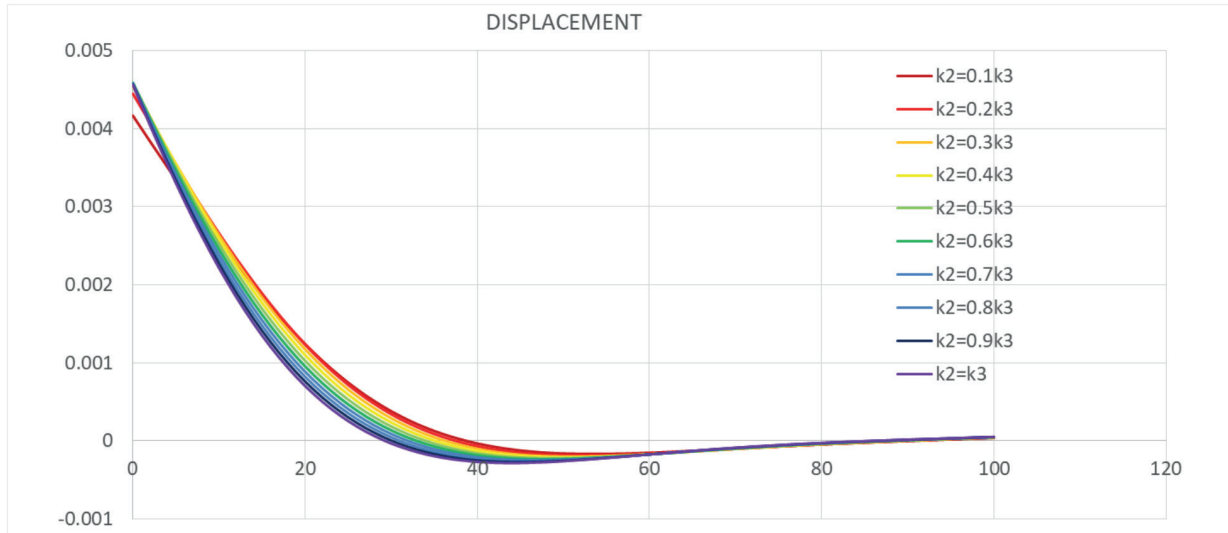
the force and the deflection is:

$$EA \frac{d^2 w}{dx^2} = N_z \tag{14}$$

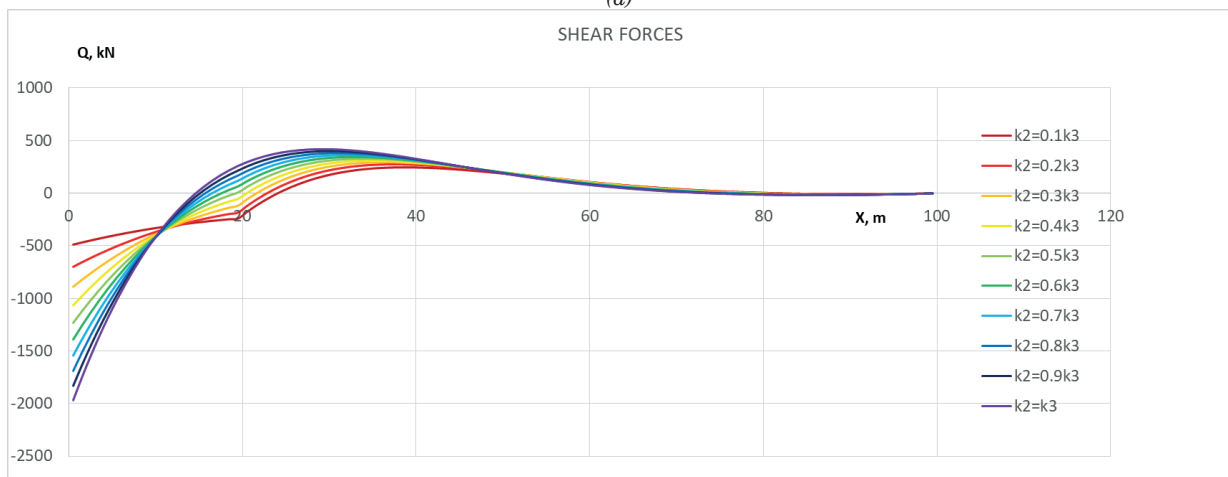
$$N_z = \pi d k_s w = 1687 \text{ kN} . \tag{15}$$

Given Equation (13) and the equilibrium equation for the right-hand cut-off part, one gets:

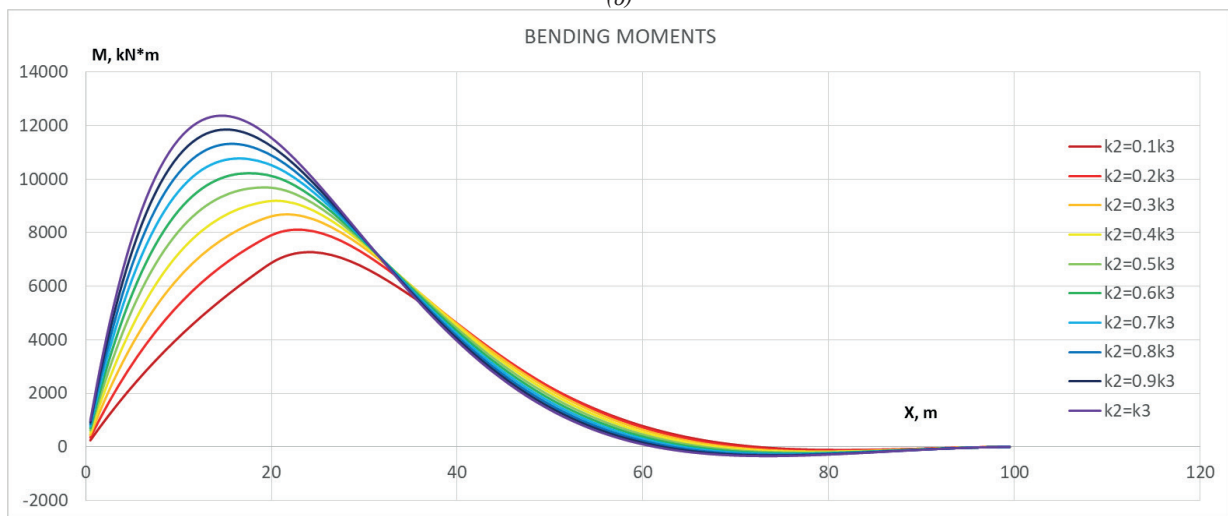
Obviously, the force at the ends of the tunnel is equal to zero. Then, one can plot the diagram internal forces  $N_z$ .



(a)



(b)



(c)

**Figure 11** Diagram of displacement (a), shear forces(b) and bending moments (c) in terms of  $k_2$  (1D scheme)

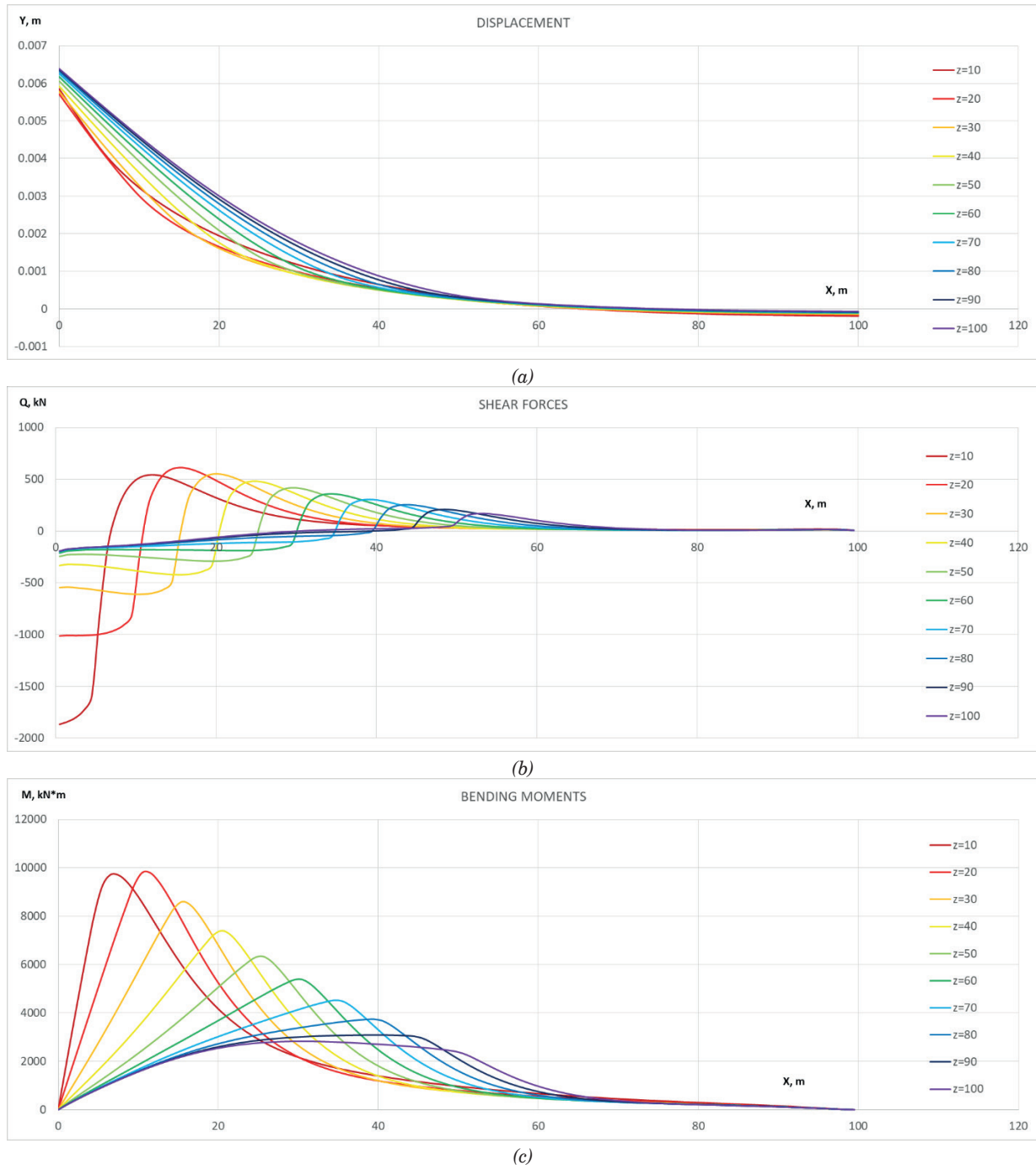


Figure 12 Diagram of displacement (a), shear forces (b) and bending moments (c) in terms of  $z$  (2D scheme)

### 3.3 Numerical simulation

For comparing the results, the previously presented models of the tunnel as beams on an elastic base with springs (1D) and model with ground given as plane strain elements (2D) are used.

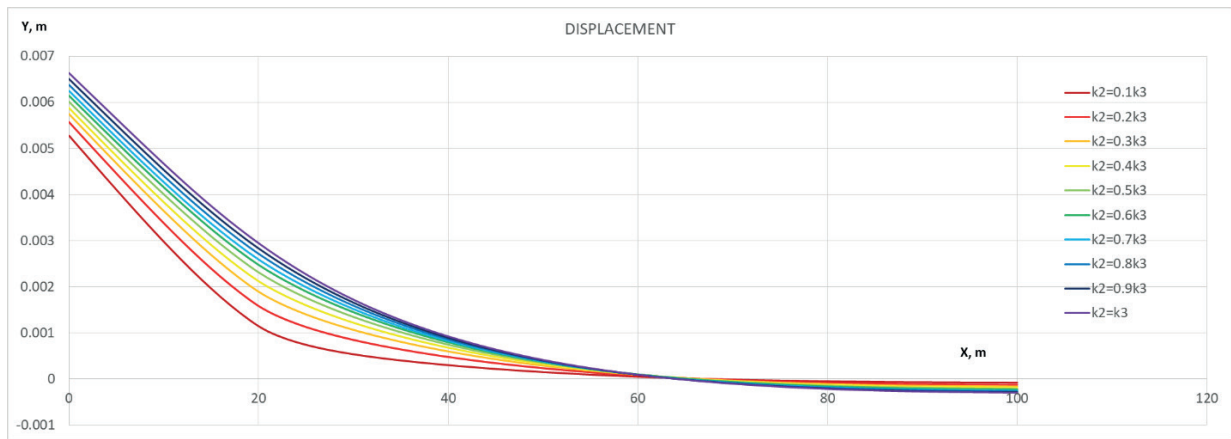
Using these models, with a difference only in the direction of the applied load (the horizontal direction instead of the vertical one). Friction was modeled like an interface elements with strength reduction factor  $R_c = 0.5$ . The results are presented in Figure 16.

### 4 Simultaneous action of the fault perpendicular and along the tunnel axis

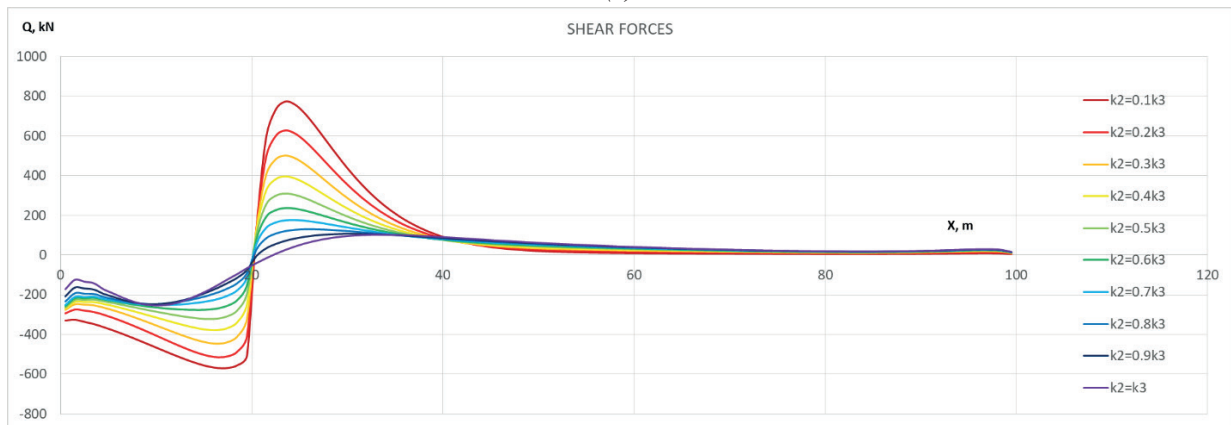
Using the introduced calculation schemes, it is possible to consider the problem combine stress-strain state of the tunnel lining (Figure 17). For clarity the 1D and 2D schemes were compare, the obtained results are presented in Figure 18.

Initial data:

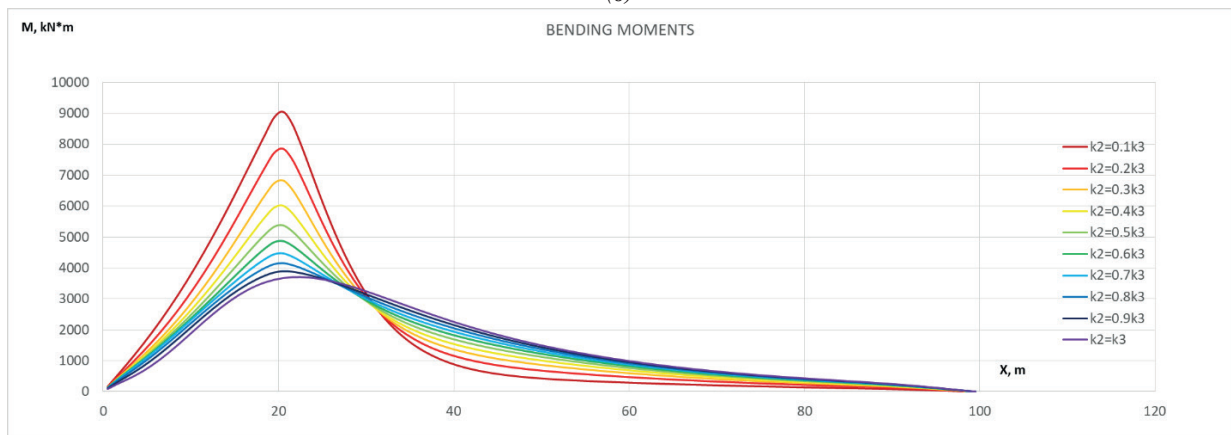
- For the 1D scheme:  
 $k_1 = 40000 \text{ kN/m}^2$ ,  $k_2 = 0.2k_1 = 8000 \text{ kN/m}^2$ ,  
 $k_3 = k_1 = 40000 \text{ kN/m}^2$



(a)



(b)



(c)

Figure 13 Diagram of displacement (a), shear forces (b) and bending moments (c) in terms of  $E_2$  (2D scheme)

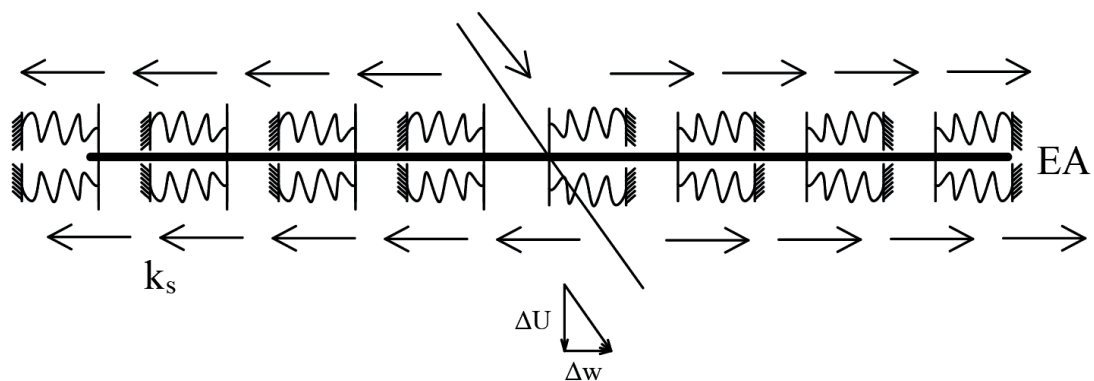


Figure 14 Calculation scheme for the case when fault is along the tunnel. cross sections axis

- For the 2D scheme to transition to another soil model, the Scott's Equation (12) was used to obtain the Young modulus for the ground blocks.
- Value of the displacement in each direction equals to 0.005 m.
- Same tunnel lining geometrical and physical parameters.
- $k_1 = 40000 \text{ kN/m}^2$ ,  $k_2 = 0.2k_1 = 8000 \text{ kN/m}^2$ ,

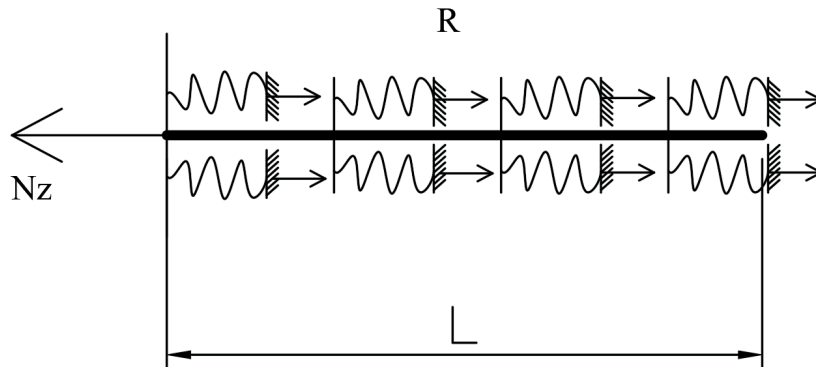


Figure 15 The right-hand cut-off part

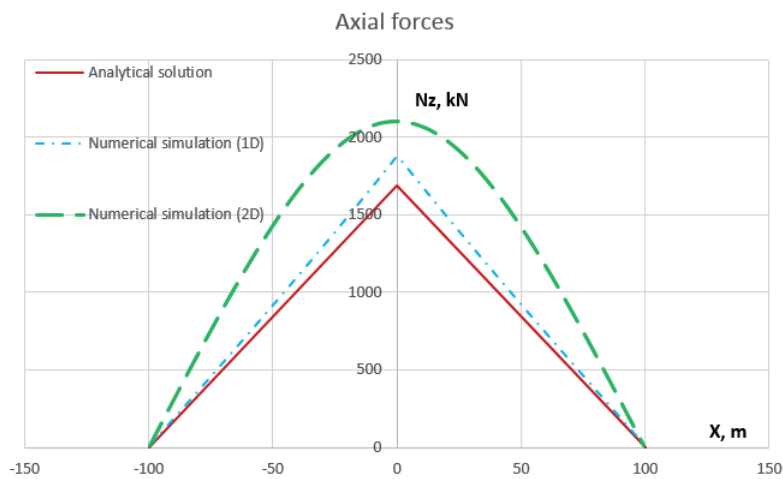


Figure 16 Results of axial forces for the silty clay subgrade

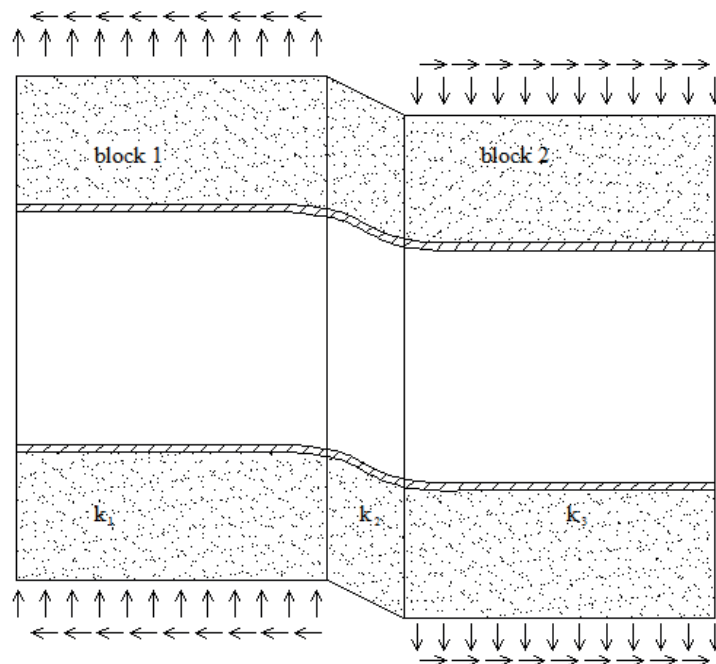


Figure 17 The combined load set

$k_3 = k_1 = 40000 \text{ kN/m}^2 \cdot 0.005 \text{ m}$  The diagrams of shear forces and bending moments have a good match, so it is acceptable to use these schemes to consider the bending and shear effects. The diagrams of axial forces have error rate of about 20%, so it means that this approach allows to estimate previous assessment of stress-strain state under the combined load set. However, the displacement diagrams have a big divergence, as well.

In the 1D scheme it is impossible to consider mass of the ground above the tunnel, moreover, to transit to the 2D scheme, an approach given by Scott (1981) was used. Obviously, these models have different stiffness of the whole system "ground-structure".

The main hypothesis is that the coefficient of the subgrade reaction for the tension-compression is a quarter of a normal coefficient of the subgrade reaction for bending and shearing actions  $k_s = \frac{1}{4}k$ . This investigation was completed for piles and for tunnels it is necessary to continue this research.

## 5 Equivalent stiffness

In all the tasks discussed above, the tunnel lining is assumed to be made of monolithic concrete with a constant cross-section. However, in engineering practice, the precast structures, made of reinforced concrete or cast iron, are also used. In this case, one needs to take into account the bending stiffness and tensile-compression stiffness, which differ depending on the geometry, type of cast iron, type of bars in the reinforced concrete, location of joints of the segmental linings, number of bolts in a specific cross-section.

### 5.1 Equivalent stiffness in tension and compression

Stiffness of the transverse joint in tension is:

$$K_j = \frac{nE_j A_j}{l_j}, \quad (16)$$

where  $n$  is the number of bolts in the cross-section area,  $E_j A_j$  - the stiffness of a bolt in tension,  $l_j$  - length of a bolt.

In compression, bolts do not carry load, so in this case equivalent stiffness is determined by the lining stiffness; however in tension the lining ring and the transverse joint link work together, so equivalent stiffness can be presented:

$$\frac{l_s}{(EA)_{eq}^T} = \frac{1}{K_j} + \frac{l_s}{E_s A_s} \text{ or } (EA)_{eq}^T = \frac{K_j E_s A_s / l_s}{K_j + E_s A_s / l_s}. \quad (17)$$

### 5.2 Equivalent stiffness in bending of the segment lining

Consider the equilibrium of a part of structure, consisting of the two halves of a ring and the transverse joint (Figures 19 and 20) with assumptions:

- 1) the width of a ring is much smaller than the tunnel length, so changes in the stress along the axis can be neglected,
- 2) the thickness of a ring is much smaller than the radius.

Equivalent stiffness of the transverse joint [4] is:

$$(EI)_j = \frac{E_s I_s l_j}{l_s} \frac{\cos^3 \theta}{\cos \theta + \left(\frac{\pi}{2} + \theta\right) \sin \theta}, \quad (18)$$

where  $I_s = \pi R^3 t$  is an axial moment of inertia of ring's cross-section area.

The rotation angle of equivalent ring's cross-section area is a sum of rotation angles of the ring and a joint, i.e.

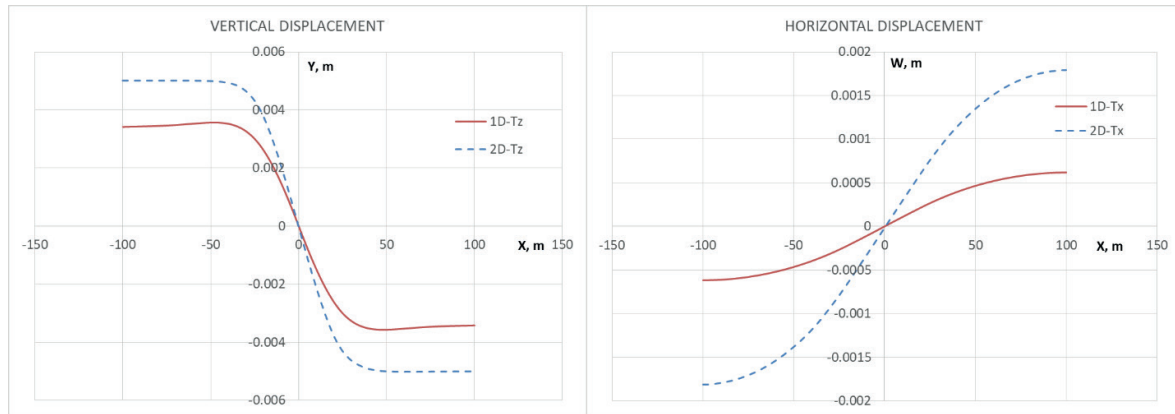
$$\frac{l_s}{(EI)_{eq}} = \frac{l_s}{E_s I_s} + \frac{l_j}{(EI)_j} \quad (19)$$

Combining Equations (18) and (19), one obtains the equivalent stiffness of the cross-section tunnel lining in bending as:

$$(EI)_{eq} = E_s I_s \frac{\cos^3 \theta}{\cos^3 \theta + \cos \theta + \left(\frac{\pi}{2} + \theta\right) \sin \theta}. \quad (20)$$

## 6 Conclusions

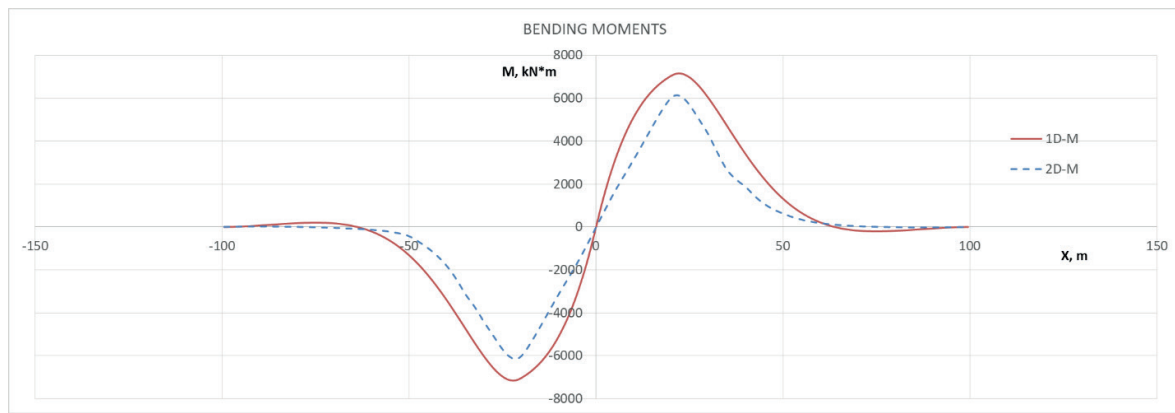
- Classical model of a beam on an elastic foundation is adopted for solving the problem of seismic and geophysics impact on the tunnel in fracture zone;
- The 1D and 2D numerical simulation models for solving the mentioned problem are introduced and verified by analytical solutions;
- Those models in complex combined loading case are tested;
- The case with equivalent stiffness of the tunnel lining is presented.



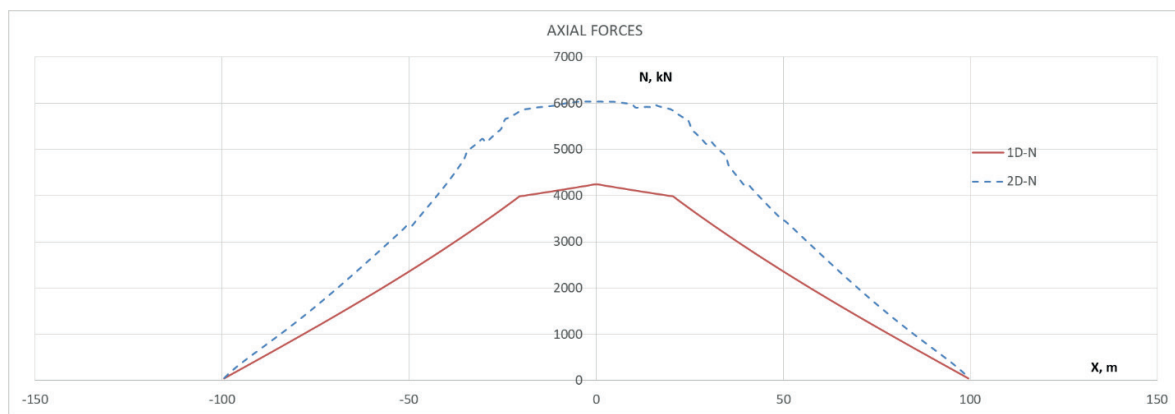
(a)



(b)



(c)



(d)

**Figure 18** The combined load set case. Diagrams of displacements (a), shear forces (b), bending moments (c) and axial forces (d) in terms of distance  $x$ .

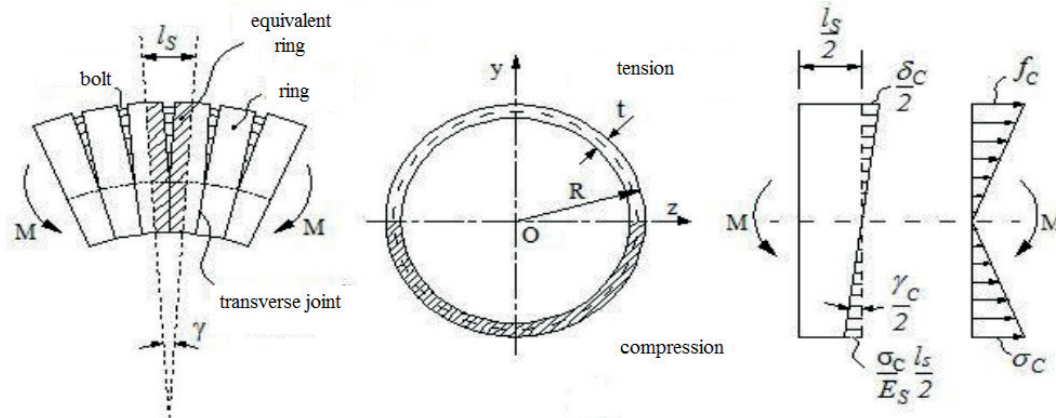


Figure 19 Stress distribution and angle displacement of the cross-section area in the ring

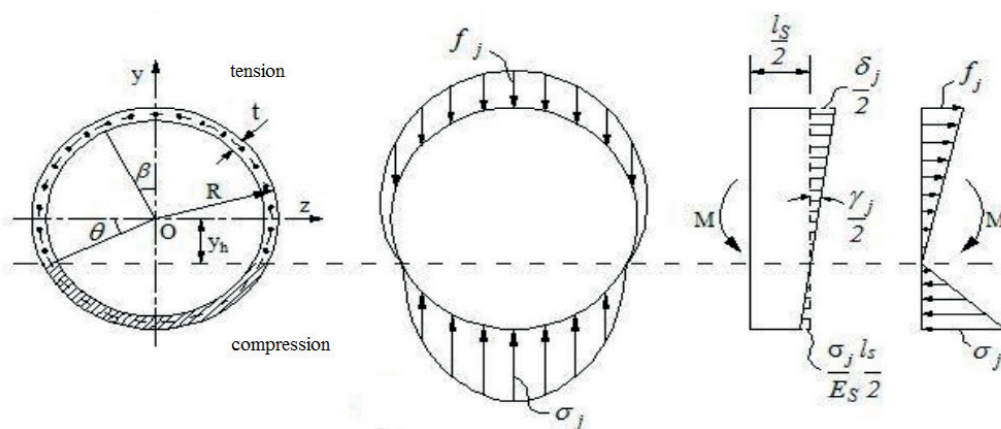


Figure 20 Stress distribution and angle displacement of the cross-section area in the transverse joint

## References

- [1] ASAKURA T., TSUKADA K., MATSUNAGA T., MATSUOKA S., YASHIRO K., SHIBA Y., OYA T. Damage to mountain tunnels by earthquake and its mechanism. *Proceedings of Japan Society of Civil Engineers (JSCE)* [online]. 2000, **2000**(659), p. 27-38. eISSN 2187-5103. Available from: [https://doi.org/10.2208/jscej.2000.659\\_27](https://doi.org/10.2208/jscej.2000.659_27)
- [2] DOWDING C.H., ROZEN A. Damage to rock tunnels from earthquake shaking. *Journal of Geotechnical Engineering Division ASCE*. 1978, **104**(2), p. 175-191. ISSN 1090-0241, eISSN 1943-5606.
- [3] KONTOGIANNI V., STIROS S. Earthquakes and seismic faulting: effects on tunnels. *Turkish Journal on Earth Sciences*. 2003, **12**, p. 153-156. ISSN 1300-0985, eISSN 1303-619X.
- [4] KURBATSKY E. Equivalent stiffness of the segment lining when bending is perpendicular to the tunnel axis / Эквивалентная жесткость сборной обделки при изгибе перпендикулярно оси тоннеля (in Russian). In: VI international Conference of Professors, Young Scientist and Students: proceedings. 2013. p. 3-6.
- [5] MAI, D. M. Seismic analysis of tunnels crossing the fault zone in elasto-plastic soils / Raschet tonneley, raspolozhennykh v uprugoplasticheskikh gruntakh, peresekayushchikh zony razloma, na seymicheskie vozdeystviya (in Russian). *Building and Reconstruction*. 2013, **1**(45), p. 19-26. ISSN 2073-7416.
- [6] SCOTT, R. F. *Foundation analysis*. New Jersey, Prentice-Hall: Englewood Cliffs, 1981. ISBN 978-0133291698.
- [7] TIMOSHENKO, S. *Strength of materials. Vol. II. Advanced theory and problems*. 3. ed. Melbourne (Florida): Krieger Publishing Company, 2002. ISBN 978-8123910772.
- [8] WINKLER, E. *On elasticity and fixity / Die Lere fon der Elastitsitet und Festigayt* (in German). Prague: Dominicus, 1867.
- [9] ZAINAGABDINOV, D., MAI, D. M. Mathematical models for automated monitoring of tunnels in fault zones / Matematicheskie modeli pri avtomatizirovannom monitoringe tonneley v zonakh razlomov (in Russian). *Modern Technologies. System analysis. Modelling*. 2013, **4**(40), p. 66-72. ISSN 1813-9108.
- [10] ZAINAGABDINOV D., MAI, D. M. Models for calculations crossing active faults tunnels / Modeli dlya rascheta tonneley, peresekayushchikh aktivnye razlomy (in Russian). *Naukovedenie* [online]. 2013, **3**, 25TBH313. Available from: <https://naukovedenie.ru/PDF/25tvn313.pdf>