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WHEEL/RAIL TANGENTIAL CONTACT STRESS EVALUATION BY MEANS OF THE MODIFIED STRIP METHOD

The article deals with the way of calculating tangential stresses over non-elliptical contact patch where Kalker's simplified method FASTSIM can be used advantageously. This method named FASTSTRIP is adapted for non-elliptical contact area calculated by means of the Strip method. This method is almost as quick as FASTSIM and the results are similar to the CONTACT results. This method may be useful for rail vehicles in track dynamics computation.

Keywords: FASTSIM, FASTSTRIP, modified Strip method, wheel/rail contact, contact tangential stress evaluation, optimized computation procedure.

1. Introduction

The rail /wheel contact relations for purposes of rail vehicles dynamics are often calculated by means of Hertz method [1] and Kalker's simplified method applied in the program code FASTSIM [2]. Kalker's variation method [3] used to be considered as an etalon for contact patch and contact stress between railway wheel and rail calculation. Normal stresses and contact patches areas are assessed with the program code NORM [3], tangential stresses and tangential forces with the program code TANG [3]. The computation with Kalker's variation method takes longer time than the computation with the simplified method. This is the reason why the variation method is not widely used for rail vehicles dynamics computation and the simplified method is preferred for this purpose. The results gained with the simplified method are partially different (but acceptable) from the results gained with the variation method. The most significant difference consists in the contact area shape and size calculated in program FASTSIM that always presupposes to be elliptical. The Strip method procedures [4] give more opportunities to solve the contact with respect to non-elliptical contact patch. Our aim is to create the calculation procedure of "FASTSIM" type - we can name it "FASTSTRIP" for calculation of stresses over a non-elliptical contact area. We derived the procedures for fast non-elliptical contact patch calculation [5] as a presupposition of tangential forces computation. The resultant values of our brand new procedure presented in this article are closer to calculation results achieved by Kalker's variation method [3], while the computation speed is similar to the computation speed

of FASTSIM [2]. The authors' initial remarks on the wheel/rail tangential contact stress evaluation by means of the modified Strip method can be found in [6] and first article in [7].

2. Prerequisites

To begin with, we calculate the moduli of shear elasticity for wheel and rail materials [3]:

$$G_1 = \frac{E_1}{2 \cdot (1 + \nu_1)}, G_2 = \frac{E_2}{2 \cdot (1 + \nu_2)},$$

$$G_{12} = \frac{2}{\frac{1}{G_1} + \frac{1}{G_2}} \quad (1)$$

where:

G_1, G_2 - moduli of shear elasticity,

E_1, E_2 - moduli of elasticity,

ν_1, ν_2 - Poisson's ratios,

The contact area assembled from strips and normal stress above the strips is calculated with the Strip method.

The output parameters from the Strip method that come into the modified procedure are:

N - number of strips,

y_i - centre of i -th strip coordinate,

y_d - half-length of the strips,

x_{di} - half-length of the i -th strip,

p_{oi} - normal stress in the middle of i -th strip,

A_{NS} - area of all strips.

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We used the modified "FASTSTRIP" method for the tangential stresses computation. Figure 1 shows the program dialog with graphical output of results.

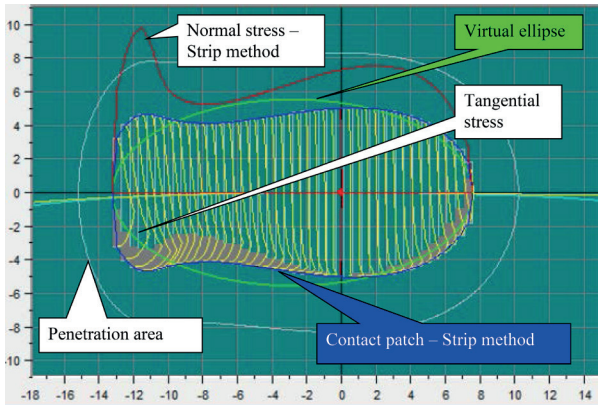


Fig. 1 Plot of AreaNORM and AreaFASTSTRIP against wheelset treads profiles lateral movement

We use this method for computation of the stresses in the non-elliptical contact patch area. Before the computation, it is needed to compute or find out the virtual ellipse parameters:

Slips s_x, s_y and spin ψ are calculated from the geometrical relations.

For constants C_{11}, C_{22}, C_{23} determination, the imaginary elliptical contact patch with the b semi-axis:

$$b = \frac{y_N - y_1}{2} + y_d \quad (2)$$

the a semi-axis:

$$a = \frac{A_{NS}}{\pi \cdot b} \quad (3)$$

and with the ellipse centre coordinate y_0 :

$$y_0 = \frac{y_1 + y_N}{2} \quad (4)$$

will be used.

For the semi axes proportion the following relation is valid:

$$D = \frac{b}{a}. \quad (5)$$

We will set C_{11}, C_{22}, C_{23} constants for the given D parameter and μ friction coefficient.

For C_1, C_2, C_3 constants the following relations are valid:

$$C_1 = \frac{9}{32} \cdot C_{11}, C_2 = \frac{9}{32} \cdot C_{22}, C_3 = \frac{3\sqrt{D}}{\pi} \cdot C_{23} \quad (6)$$

We determine the number of splitting up the strips in the longitudinal direction:

$$n_x = \text{Int}(8 \cdot a) \quad (7)$$

The tangential forces T_x, T_y and the spin moment M_z are set to be zero at the beginning.

3. Mathematical computational algorithm

The mathematical computational algorithm is schematically depicted in the flow chart in Fig. 2.

3.1 Calculation at the i-th strip (A):

For the i -th strip position the coordinate in the imaginary ellipse area is as follows:

$$y_{ei} = \frac{y_i}{b} \quad (8)$$

and for its width the following holds:

$$y_{ed} = \frac{2 \cdot y_d}{b}. \quad (9)$$

For tangential maximum stress the relation mentioned below holds:

$$T_0 = \mu \cdot p_{0i}, \quad (10)$$

where:

μ is the friction coefficient.

We will calculate the following constants:

$$C = \frac{G_{12}}{T_0}, U_x = C \cdot C_1 \cdot s_x, U_y = C \cdot C_2 \cdot s_y, \quad (11)$$

$$F_x = C \cdot C_3 \cdot b \cdot \psi, F_y = C \cdot C_3 \cdot a \cdot \psi$$

For the half-length of the strip the following relation holds:

$$a_{ei} = \frac{x_{di}}{a}. \quad (12)$$

For a calculating step we have:

$$\delta_x = \frac{a_{ei}}{n_x} \quad (13)$$

and for the area element:

$$A_e = y_{ed} \cdot \delta_x \quad (14)$$

For the strip slip in the x -axis direction we have:

$$s_{xi} = U_x - F_x \cdot y_{ei}. \quad (15)$$

3.2 Calculation over the length of i-th strip (B):

The p_x, p_y tangential stresses are set to be zero at the beginning of the calculation over the strip length.

The x_e coordinate is being changed in interval

$$\left\langle a_{ei} - \frac{\delta_x}{2}, -a_{ei} + \frac{\delta_x}{2} \right\rangle \text{ with a step of } \delta_x.$$

For the current slip in the y direction following holds:

$$s_{yi} = U_y + F_y \cdot \frac{(a_{ei} + x_e)}{2}. \quad (16)$$

For tangential stresses over the area element with coordinates (x_e, y_{ei}) following relation holds:

$$p_x = p_x - s_{xi} \cdot (a_{ei} - x_e), p_y = p_y - s_{yi} \cdot (a_{ei} - x_e) \quad (17)$$

Then the stress amplitude and maximum feasible amplitude ratio will be calculated

$$p = \frac{\sqrt{p_x^2 + p_y^2}}{a_{ei} - x_e}. \quad (18)$$

If $p > 1$, then:

$$p_x = \frac{p_x}{p}, p_y = \frac{p_y}{p}. \quad (19)$$

We will calculate the T_{ex}, T_{ey} tangential forces and M_{ez} spin moment for an area element.

$$\begin{aligned} T_{ex} &= p_x \cdot A_e, T_{ey} = p_y \cdot A_e, \\ m_{ez} &= (p_x \cdot y_{ei} - p_y \cdot x_e) \cdot A_e \end{aligned} \quad (20)$$

These forces and spin moment are added to T_x, T_y forces and M_z spin moment:

$$T_x = T_x + T_{ex}, T_y = T_y + T_{ey}, M_z = M_z + M_{ez} \quad (21)$$

Having completed the calculation over all the strips, the T_x, T_y forces and the M_z spin moment will be divided by the $T_z = \frac{\pi}{2}$ constant.

$$T_x = \frac{T_x}{T_z}, T_y = \frac{T_y}{T_z}, M_z = \frac{M_z}{T_z}. \quad (22)$$

To obtain true values of T_x, T_y and M_z spin moment, the values calculated from (22) have to be multiplied by the constant T :

$$T = N \cdot \mu, \quad (23)$$

$$T_x = T_x \cdot N \cdot \mu, T_y = T_y \cdot N \cdot \mu, M_z = M_z \cdot N \cdot \mu \quad (24)$$

where:

μ is a friction coefficient,

N is a normal force.

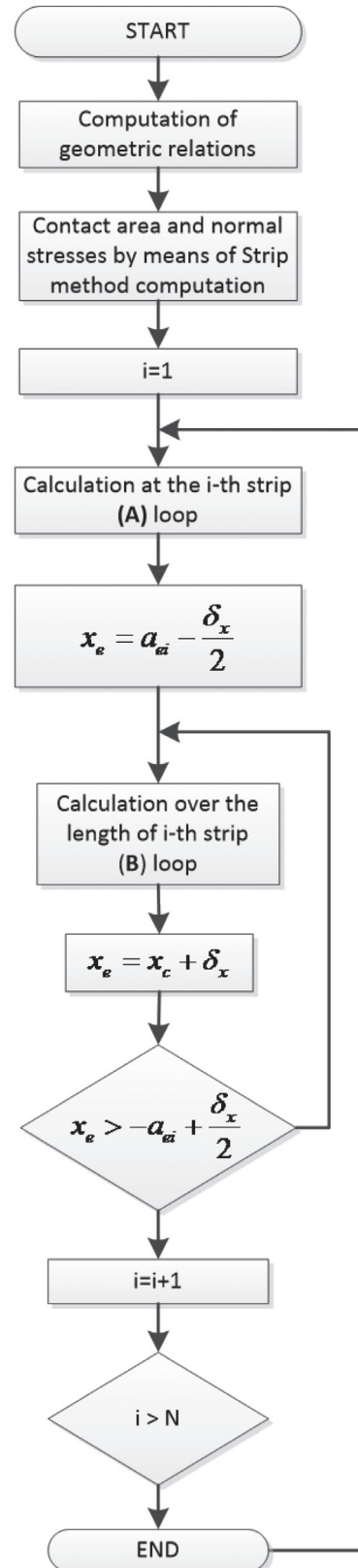


Fig. 2 Flow chart of the procedure

4. Results and validation

We analyzed the contact patch area, contact stress between the wheel equipped by S1002 tread profile and UIC60 rail head profile inclined by 1:40. The lateral shift is in interval of (cca -5mm to 5mm).

4.1 Input parameters

The wheel force is $Q = 100.000N$. We used our fast strip method [5] for the computation of contact patches and contact stresses and we compared our results with the results obtained by Kalker's Contact-NORM method [3].

In Table 1 are summarized input parameters: y_w [mm] is lateral shift of wheels profiles over the rail heads profiles, F_n [N] is a normal force, $\tan(\text{Gama})$ is the value of Tangent Gamma function that is the contact area angle tangent in the contact point. Gama [rad] is the same angle expressed in radians. S_x and S_y are the slips (creepages in x and y directions) and Φ is expressed in [rad/mm]. The difference with the FASTSIM

method is that the computation is executed along the strip separately, regardless of the strip size - whether it is smaller or longer than the virtual ellipse border.

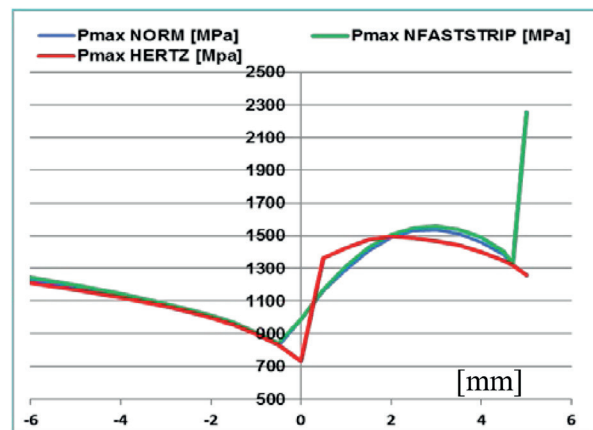


Fig. 3 Plot of P_{max} CONTACT-NORM, P_{max} NFASTSTRIP and P_{max} HERTZ against wheelset treads profiles lateral movement

Contact input parameters

Table 1

y_w	F_n [N]	$\tan(\text{Gama})$	Gama [rad]	S_x [-]	S_y [-]	Φ [rad/mm]
-5	100004	-0.008948	-0.00894776	0.001645	0	0.000019
-4	100004.7	-0.009655	-0.00965470	0.001295	0	0.000021
-3	100006.0	-0.010960	-0.01095956	0.001054	0	0.000024
-2.5	100007.1	-0.011927	-0.01192643	0.000953	0	0.000026
-2	100008.6	-0.013144	-0.01314324	0.000859	0	0.000029
-1.5	100010.9	-0.014776	-0.01477492	0.000773	0	0.000032
-1	100014.2	-0.016839	-0.01683741	0.000684	0	0.000037
-0.5	100019.5	-0.019748	-0.01974543	0.000595	0	0.000043
0	100029.0	-0.024072	-0.02406735	0.000000	0	0.000052
0.5	100222.8	-0.066581	-0.06648288	-0.000595	0	0.000145
1	100249.4	-0.070613	-0.07049599	-0.000684	0	0.000153
1.5	100282.8	-0.075148	-0.07500702	-0.000773	0	0.000163
2	100317.3	-0.079742	-0.07957362	-0.000859	0	0.000173
2.5	100360.8	-0.084998	-0.08479419	-0.000953	0	0.000184
3	100411.8	-0.090812	-0.09056359	-0.001054	0	0.000197
3.5	100472.2	-0.097318	-0.09701251	-0.001166	0	0.000210
4	100550.2	-0.105034	-0.10465029	-0.001295	0	0.000227
4.5	100652.9	-0.114436	-0.11394035	-0.001449	0	0.000247
4.7	100703.2	-0.118818	-0.11826354	-0.001519	0	0.000256
5	100799.7	-0.126724	-0.12605211	-0.001645	0	0.000273

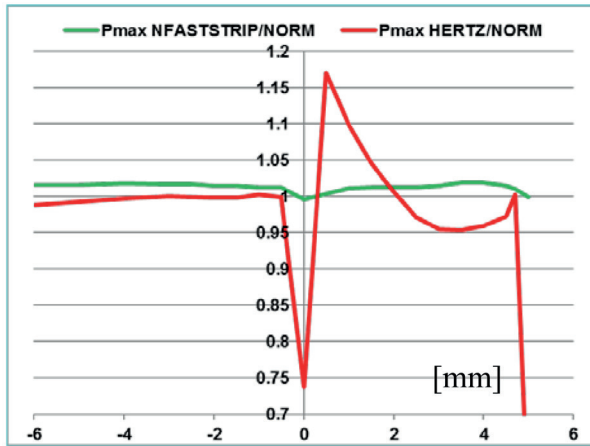


Fig. 4 Plot of P_{max} NFASTSTRIP/NORM and P_{max} HERTZ / NORM [-] proportional comparison of evaluated quantities

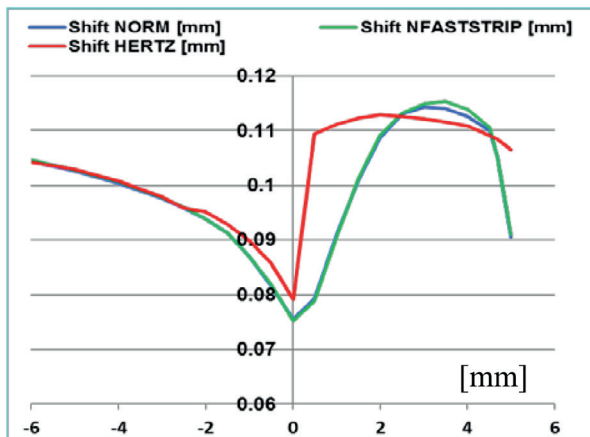


Fig. 5 Plot of Shift NORM, Shift FASTSTRIP and Shift HERTZ against wheelset tread profiles lateral movement

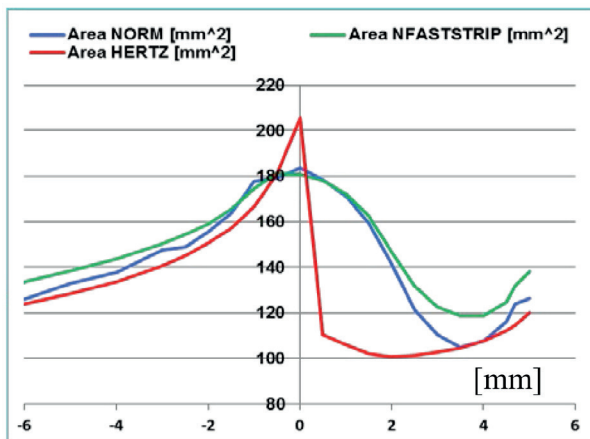


Fig. 6 Plot of Area NORM, Area FASTSTRIP and Area HERTZ against wheelset tread profiles lateral movement

4.2 Results of tangential stresses calculation

Tangential stresses, T_x , T_y forces and M_z moment for separate strips are computed by means of the T_x , T_y and M_z computed by "CONTACT-TANG", "FASTSIM" and "FASTSTRIP" methods for the purpose of comparison.

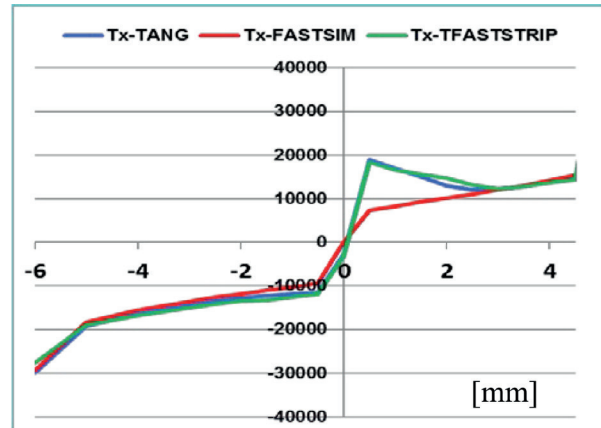


Fig. 7 Plot of T_x [N] against wheelset tread profiles lateral movement

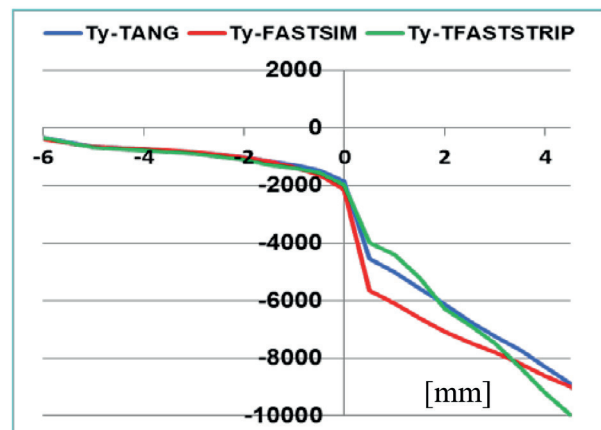


Fig. 8 Plot of T_y [N] against wheelset tread profiles lateral movement

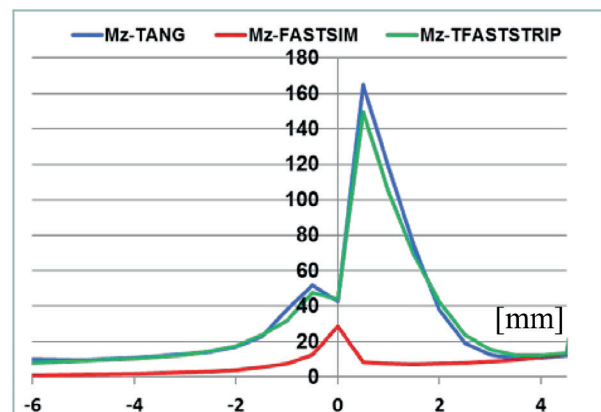


Fig. 9 Plot of M_z [N.m] against wheelset tread profiles lateral movement

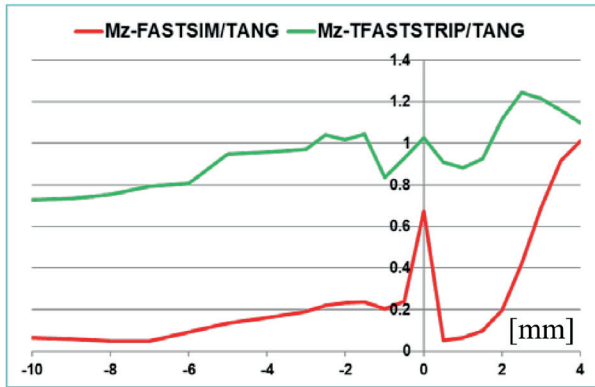


Fig. 10 Plot of M_z [N.m] - Proportional comparison of evaluated quantity

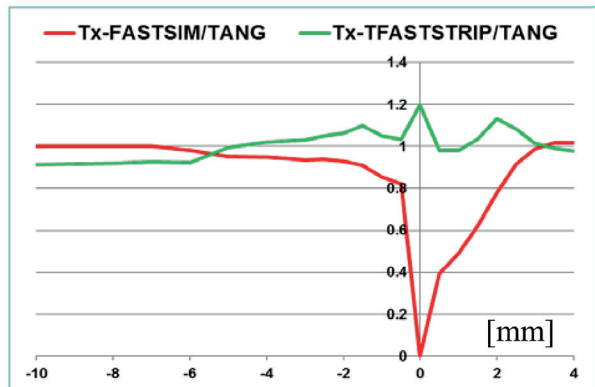


Fig. 11 Plot of T_x [N] - Proportional comparison

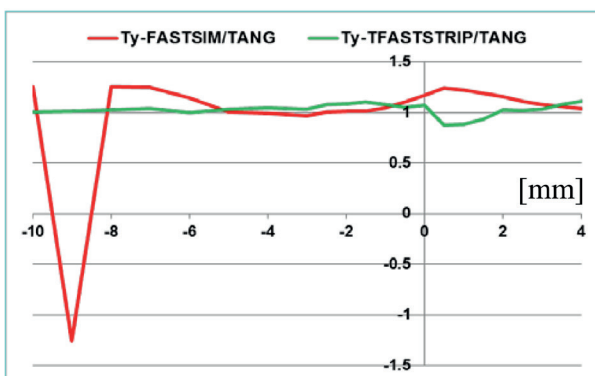


Fig. 12 Plot of T_y [N] - Proportional comparison of evaluated quantity

5. Conclusions

Our aim is to create the calculation procedure of "FASTSIM" type. We named this procedure for calculation of stresses over non-elliptical contact area "FASTSTRIP". The result values are closer to the results achieved by Kalker's variation method results and the computation speed is similar to the computation speed of FASTSIM. This method is adapted for a non-elliptical contact area calculated by means of the Strip method [5]. This method utilizes the FASTSIM theory [2] as a calculation engine for tangential stress assessment. The calculation procedure is outlined in Fig. 2 in which a flowchart with two program loops is illustrated. These loops are in detail described in the part "Mathematical model". Results and validation follow. In Table 1 are some input parameters, Figs. 3, 4, 5 and 6 compare the results gained by means of NORM [3] and our calculation procedure [5] shift, area and pmax. They express the reality that the ground input parameters for tangential forces calculations are mutually very close.

Figures 7, 8, 9 and 10 give results of tangential stresses calculation for input parameters. The curves of dependencies (T_x , T_y , M_z) calculated with TANG [3] and FASTSTRIP are shown in graphs. For better resolution are these curves shown in Figs. 11 and 12 as comparative proportional curves. The meaning or importance of the procedure FASTSTRIP for us or anybody who writes his/her own code is in the fact that this procedure can be implemented into the code for computation of rail vehicles dynamics offering the advantage of fast computations. Of course, the field may be analysed from other points of view [8, 9 and 10] where multi-software platforms are presented. The field of wheel /rail contact task is very close to the computational analysis of contact stress distribution in the case of mutual slewing of roller bearing rings [11 and 12], or from the more sophisticated field [13, 14 and 15].

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