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ESTIMATION OF SHEAR STIFFNESS OF INTERLAYER CONNECTION IN TWO-LAYER COMPOSITE BEAMS BASED ON ANALYSIS OF NATURAL FREQUENCIES

The paper shows possibilities of testing composite two-layer beams with flexible connectors (like bolts in steel-concrete beams and nails in wooden-concrete ones) offered by analysis of their natural frequencies. It enables an estimation of shear compliance for the interlayer connection if Young's moduli of layers are known thanks to finding the global minimum of function of errors between model and measured natural frequencies of beam. The considerations are illustrated by the results of our own laboratory-tests.

Keywords: *Composite beam, compliant shear connection, natural frequency, laboratory tests.*

1. Introduction

Composite structures like steel-concrete and wooden-concrete ones with compliant shear connection of their layers become more and more popular systems in civil engineering. First of all, a considerable increase of their load capacity and stiffness resulting from combining their components should be appreciated for this type of structures. Moreover, the problem of torsional-flexural buckling of more flexible steel (or wooden) profiles is eliminated thanks to connecting them with the rigid upper slab [1]. It should be stressed that these kinds of beam structures work correctly if the shear connection between the ferroconcrete slab and steel (wooden) bar elements shows enough low compliance. This characteristic is investigated, first of all, by means of push-out tests (for example [2]) or measurements of beam deflection under static load (for example [3]). The paper shows another method of estimating the shear compliance for the beam interlayer joints based on analysis of their natural frequencies which may be a complementary measuring method of the enumerated ones. The problem is illustrated by the experimental results obtained thanks to our own tests carried out in the laboratory-scale.

2. Mathematical description of the problem

Assuming the simplest linear elastic model for a two-layer composite beam with a compliant interlayer connection (for example [3]) the system of differential equations for functions: w, $u_{(1)}$ and $u_{(2)}$ (vertical displacement of centre line of the beam and horizontal displacements of centre lines of the lower and upper layer, respectively) may be written as follows (with neglected damping) (for example [4]):

$$\left\{\begin{array}{lll} w^{IV}\left(E_{(1)}I_{(1)}+E_{(2)}I_{(2)}\right)-ke^2w^{II}-ke^2w^{I} & +keu^{1}_{(1)}+k^{1}eu_{(1)} & -keu^{1}_{(2)}-k^{1}eu_{(2)} =-q-\left(\mu_{(1)}+\mu_{(2)}\right)\ddot{w} \\ \quad kew^{I} & +E_{(1)}A_{(1)}u^{II}_{(1)}-ku_{(1)} & +ku_{(2)} =\mu_{(1)}\ddot{u}_{(1)} \\ \quad -kew^{I} & +ku_{(1)} & +E_{(2)}A_{(2)}u^{II}_{(2)}-ku_{(2)} =\mu_{(2)}\ddot{u}_{(2)} \end{array}\right. \qquad (1)$$

where: k – stiffness of the shear connection (in [$N \cdot m^{-2}$]); $E_{(i)}$ – Young's modulus of layer (i); $I_{(i)}$ – moment of inertia of layer (i); $e=½(h_{(1)}+h_{(2)})$ where $h_{(i)}$ is a cross-sectional height of layer (i); $\mu_{(i)}$ – density over the length of layer (i); i=1,2 – index of beam layer; $(\ )^{I}$, $(\ )^{II}$, $(\ )^{IV}$ – the first, second and fourth derivatives along the x-axis; $(\ddot{\ })$ – the second time-derivative, q – external vertical distributed load. In order to solve the above system for a specific problem it has to be completed by the proper boundary and initial conditions. Using the Finite Element Method (FEM) or Finite Difference Method (FDM), we can express the system (1) generally in the matrix form:

$$\mathbf{Ku = P - B\ddot{u}} \qquad (2)$$

where: **K** – stiffness matrix, **u** – vector of node displacements, **P** – vector of forces (three terms in the node equations), **B** – inertia matrix. In order to formulate an eigenvalue problem for this case we assume in the equation (2) **P=0** and $\mathbf{u}=\mathbf{u}_0 \sin(\omega t+\theta)$ (for example [5]). Then we can obtain the following equation for eigenvalues λ_i of matrix $\mathbf{KB}^{-1}$ (and the same for natural angular frequencies ω_i of the beam):

$$\begin{aligned}&\left(\mathbf{K}-\omega^2\mathbf{B}\right)\mathbf{u}_0 sin\left(\omega t+\theta\right)= \\ &=\mathbf{0}\rightarrow \det\left(\mathbf{KB}^{-1}-\lambda\mathbf{I}\right)=0\rightarrow\lambda_i=\omega_i^2\end{aligned} \qquad (3)$$

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where: $\mathbf{I}$ – unit matrix, $\mathbf{0}$ – zero vector, $\mathbf{u}_0$ – vector of free vibration amplitudes, t – time. If natural frequencies of the real combined beam are known from measurements (thanks to the Fourier analysis of accelerations at chosen points in the real structure excited to test vibrations) then it is possible to estimate its stiffness k finding the minimum of the following exemplary error functions:

$$F(k)=\sum_{i=1}^{n}\left|\frac{\omega_{i(\text{measurement})}-\omega_{i(\text{model})}(k)}{\omega_{i(\text{measurement})}}\right|$$

or (4)

$$F(k)=\sum_{i=1}^{n}\left(\frac{\omega_{i(\text{measurement})}-\omega_{i(\text{model})}(k)}{\omega_{i(\text{measurement})}}\right)^2$$

where: $\omega_{i(\text{measurement})}$ – measured i-th natural angular frequency for the real structure, $\omega_{i(\text{model})}$ – i-th natural angular frequency calculated basing on the assumed model, n – number of the first natural frequencies taken into considerations.

3. Experimental results in the laboratory-scale

To illustrate the measuring possibilities offered by the free vibration analysis in the discussed scope, the experimental tests were carried out on cantilever beams in the laboratory-scale (at the temperature 20±2°C). The model of two-layer bar with sheared joint was made from two plexiglass layers 1.5m long of rectangular cross-sections (bxh=~40mm x 20mm) connected by the adhesive double-sided tape on the sides 40mm wide. The used plexiglass was characterised by the following parameters: dynamic Young's modulus E=3.99GPa, bulk density ρ=1174kg/m^3. The tape connection was used in the model to simulate the way of work of a real sheared joint.

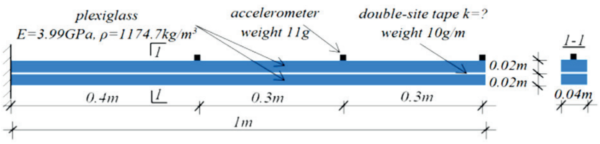

Fig. 1 The static scheme of plexiglass composite laboratory cantilever beam

The prepared two-layer bar was restrained on the solid steel element using the clamps so as to create the cantilever beam 1m long and three accelerometers (PCB 333B52 of external dimensions 11mm x 11mm x 11mm and weight ~11g with the connecting cables) were attached to the upper side of element as shown in Fig 1. Next, the cantilever was excited to vibrations by impacts applied to the lower side of element just under the accelerometers three times at each point. The accelerations were recorded on the PC computer using the software DASYLAB 10.0. An exemplary record of acceleration, which was obtained for the unbounded end of cantilever, is presented in Fig. 2. Using the Fourier transform for all the records it was found that the mean values of the first two natural frequencies were equal to:

$$f_1 = 10.43Hz,\ f_2 = 68.69Hz \tag{5}$$

Next, basing on the above values of frequencies, the minimum of function (4$_2$) (for n=2) was found by a direct search of domain for the physically possible solutions. The values for $\omega_{i(\text{model})}$ needed in the calculations were obtained by means of the own computer program written in the Matlab environment in which the eigenvalue problem, as defined by the equation (3), was solved using FDM. The diagram of function F vs. stiffness k is presented in Fig. 3. It can be noticed that one minimum was obtained in the analysed interval and it is situated at the value of stiffness equal to ~2·10^8 Pa.

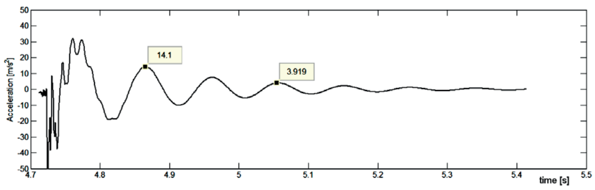

Fig. 2 The exemplary acceleration record at the unbounded end of cantilever beam

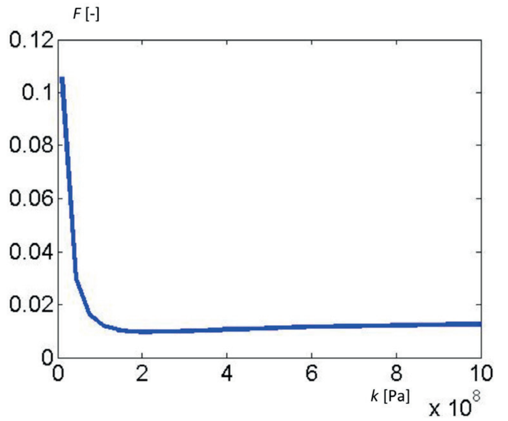

Fig. 3 The error function (4$_2$) vs. shear stiffness for n=2 in the case of tested cantilever

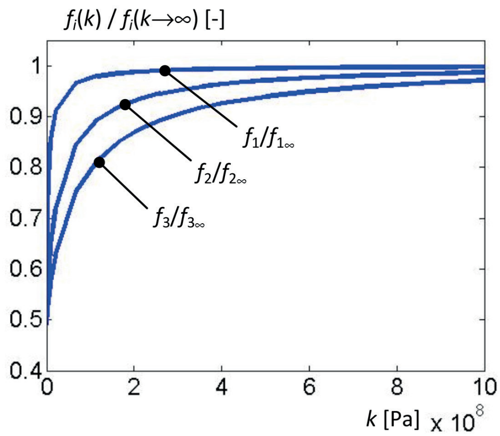

Fig. 4 First three natural frequencies vs. shear stiffness k for the data corresponding to the combined cantilever of scheme as shown in Fig. 1. The values of frequencies are normalised to their values at k→∞

Basing on these measurements it was also found that the mean value of fraction of critical damping ξ was equal to ~0.1 for the first mode of free vibrations. It is worth mentioning that the fraction ξ for the non-combined plexiglass cantilever 1m long of cross-sectional dimensions *b*x*h*=~40mm x 20mm was equal to ~0.04 which was measured by the authors in the same way as described above. The considerable increase of damping in the case of combined model was caused by viscous properties of the tape joint and introducing the mechanism of structural damping into the model in this way. However the damping for the two-layer cantilever characterised by the fraction ξ=~0.1 could not cause considerable errors in estimating the values of natural frequencies (for example [5]).

In order to show how a possible selection of number of the first natural frequencies taken into consideration (according to the pattern (4)) may influence the accuracy of results, the changes of the first three ones vs. shear stiffness k are presented in Fig. 4. The diagram was made for the data corresponding to the combined cantilever beam used in the tests described above and the values of frequencies were normalised to their values at $k \rightarrow \infty$. It can be noticed that the stiffness k can be determined more precisely if more natural frequencies are taken into account especially for its higher and low values. For example, taking only one frequency f_1 in the expressions (4) their global minimum may be found with a considerable error if input data are noised because an increase of frequency, related to a big increase of stiffness k, is very small starting from a certain value for k (in the analysed diagram approximately at $k=10^8$ Pa). The same goes for the next frequencies, but suitably at higher values of stiffness (in the analysed diagram approximately at $k=3\cdot10^8$ Pa for f_2 and $k=5\cdot10^8$ Pa for f_3). This fact may limit considerably the possibilities of application for the method if the number of the first measured frequencies is also limited due to the used equipment and measuring conditions. Basing on Fig. 4 one can state also that it should amount to 2 minimally.

4. Conclusions

The combined structures (especially two-layer beams) are more and more popular and willingly used in civil engineering applications because of their optimal use of materials with keeping required stiffness and load capacity. That is why laboratory- and non-destructive test methods should be intensively developed in this range, too. The method discussed in the work is based on the analysis of natural frequencies. It is investigated by the authors at the presented stage, first of all, from the point of view of its application in measurements of interlayer shear stiffness for combined beams in the laboratory conditions as a comparative method for the adequate push-out tests (for example [2]). The method, as formulated in the work, may be used in practice for diagnostic purposes under condition that a tested structural element can be described by a simple elastic beam model. Otherwise, it needs more advanced geometrical and physical models and software. The presented considerations illustrate also the fact that dynamic characteristics of layer structures may be determined with considerable errors if the problem of slip in their sheared interfaces is neglected.

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