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INVESTIGATION ON CRACKS CREATION AND PROPAGATION IN CONCRETE SLAB OF COMPOSITE BEAMS

Analytical and experimental research have illustrated initiation of cracks in concrete deck immediately after its casting and placing on the upper flange of steel beams, especially prior to application of additional permanent load on composite element. Not only value of concrete shrinkage strain may influence creation and propagation of cracks in concrete. However ratio of deck reinforcement, cross-sectional area of reinforcement steel and strength of concrete can modify the process. Moreover, the ratio of steel beam flexural stiffness and flexural stiffness of concrete deck can be significant. Important shrinkage strains can occur at the lower concrete surface in the steel and concrete interface. Parametrical studies have provided the relationships illustrating impact of above parameters on crack aspect. The aim of the paper is to provide further details.

1. Introduction

Steel-concrete composite bridges are nowadays considered to be an appropriate structural system for the bridge engineering (Fig. 1). The innovative systems using concrete deck can act both as a roadway resisting local traffic loads and as an integral part of the bridge girders or trusses. In such a design, the concrete slab may act also as an integral part of the compression flanges of the stringers, and cross-beams. The contribution presents problems with rheological effects estimation and concrete cracking consideration, which can be found in the design of especially continuous type composite girders of the steel and concrete. Detail requirements for advanced design of concrete members of these composite steel and concrete structures as outputs of our investigation are given in graphic form.

Fig. 1 View of a continuous composite bridge concrete deck

2. Failure of the concrete slab by cracking

Composite slab of steel and concrete structural members as shown in Fig. 2 can be affected by tensile cracking induced by shrinkage. The strength degradation may be significant, if there were not enough reinforcing bars crossing the planes of cracking. Shrinkage of the concrete element depends on the environment and the constituents of the concrete. Corresponding strains can reach values accessing of 500 microstrains. Furthermore, shrinkage is time-dependent, and therefore the forces that are created will cause creep. Longitudinal strains can be induced in composite beam due to thermal gradients. The changes in strains are the similar as for shrinkage. The tensile cracking can also be initiated by effects of concrete hydration.

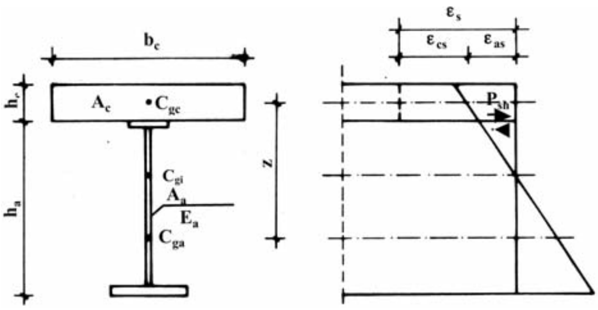

Fig. 2 Dimensions, forces in cross-section and longitudinal layout

The problem of tensile failure of the slab has not been subject of any extra fundamental investigation, because this question was

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considered only subsidiary. The main attention is generally paid to the ultimate strength analyses. Design, analysis and detailing of the bridge concrete deck are actually just in general given in Eurocode 4 [1]. Primary, the slabs are required to be designed to transmit in-plane forces as well as bending moments and shears. Where composite action becomes effective as concrete hardens, effects of heat of hydration of cement and corresponding thermal shrinkage should be taken into account during the construction stage in areas where tension is expected. Specific measures should be provided to limit the effects of heat of hydration of cement. For simplification a constant temperature difference between the concrete section and the steel beam as concrete cooler can be assumed for the determination of the cracked regions. Unless a more accurate method is used, minimum reinforcement area for the slabs of composite beams is essential for limitation of crack width.

Factor specifying capacity of composite steel and concrete beams to prevent slab cracking due thermal effects can be expressed by the ratio of cross-sectional area of the structural steel beam A_a to cross-sectional area of concrete slab A_c, i.e. $\beta = A_a/A_c$. In fact this factor can be only approximate, because the beam shape or proportion of the flanges is not taken into account.

Based on results of the investigation [2, 3, 4], it can be supposed that thermal stresses created by effects of hydration of cement can be neglected for the values of ratio $\beta \leq 0.5$. Moreover, this effect occurs only during concrete casting and its curing, when magnitude of concrete modulus is rather smaller. Contraction of the concrete through shrinkage is of prime importance for tensile cracking in the case of a composite beam design. These short time-varying deformations of concrete affect greatly serviceability limit state. In hogging region, cracks due to concrete shrinkage during the hydration process have also a significant effect on the ultimate flexural capacity of composite beams. The magnitude and rate of development of the shrinkage strain depend on such characteristics as the relative humidity, temperature, mix proportions as well as shape and size of members. With time increasing, the rate of shrinkage decreases and the shrinkage strains approach its limit value ε_{sh} at the period obviously 28 days.

3. Stresses and strains in concrete slab produced by shrinkage

In the absence of shear connection, the concrete shrinkage in the sagging region would produce the contraction of the slab and slip at the steel-concrete interface. In reality, the shear connectors resist this lack of slip. Contraction of the concrete through shrinkage will induce deflections and flexural stresses in the steel that are in the same direction as those induced by gravity loads. In order to prevent slip and hence the lack of fit, an axial force P_{sh}, shown in Fig. 2, has to be applied at the steel-concrete interface. This force acts at eccentricities from the shear connection plane to the centroids of both structural parts.

As the force in the shear connection induced by shrinkage acts on the concrete rectangular slab at the vertical distance $h_c/2$

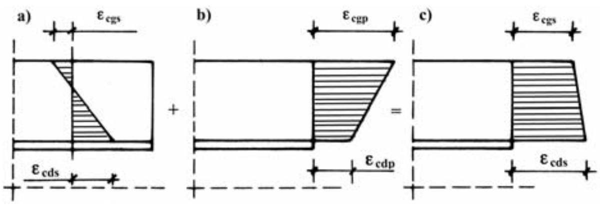

Fig. 3 Strains at the top and bottom fibre of concrete slab

from the plane of application of force P_{sh} to the slab centroid concerned, the strains at the top and bottom fibre of concrete slab in Fig. 3 are given from elementary mechanics by

$$\varepsilon_{cgs} = -\frac{2P_s}{E_c b_c h_c}, \quad \varepsilon_{cds} = \frac{4P_s}{E_c b_c h_c}. \tag{1}$$

where E_c is modulus of elasticity for concrete, b_c width and h_c thickness of the concrete slab.

The axial force in the concrete element $N_c(t)$ as well as the axial force in the steel element $N_a(t)$, which are generally time-dependent, should be equal to the axial force P_{sh}

$$P_{sh} = P_{sh}(t) = N_c(t) = N_a(t) \tag{2}$$

with $N_c(t)$ and $N_a(t)$ as axial forces variable in time t acting in concrete slab and steel beam at the level of the shear connection interface.

The force corresponding to shrinkage strain of concrete $\varepsilon_s(t)$ applied at the interface plane (Fig. 2) is given by:

$$N_c(t) = \varepsilon_s(t)\, E_c(t)\, A_c,\ N_a(t) = \varepsilon_{as}(t)/\delta_a \text{ and}$$

$$\varepsilon_{cs}(t) + \varepsilon_{as}(t) = \varepsilon_s(t). \tag{3}$$

where $\varepsilon_s(t)$ is value of shrinkage strain, $\varepsilon_{as}(t)$ conventional strains produced by steel beam at the level of concrete slab neutral axis, $\varepsilon_{cs}(t)$ strains in concrete caused by steel beam prevention to the concrete shrinkage, $E_c(t)$ modulus of elasticity for concrete, A_c cross-sectional area of concrete, z distance between the centroid of concrete slab and the centroid of the beam steel section, E_a modulus of elasticity of structural steel, A_a cross-sectional area of the structural steel section, I_a second moment of area of the structural steel section and conventional factor of flexural stiffness of the steel beam section can be expressed by

$$\delta_a = \frac{1}{E_a A_a} + \frac{z^2}{E_a I_a}. \tag{4}$$

For the strains, the following relationships can be derived:

$$\varepsilon_{as}(t) = \frac{\delta_a}{\delta_a + \delta_c}\varepsilon_s(t) \text{ and } \varepsilon_{cs}(t) = \frac{\delta_c}{\delta_a + \delta_c}\varepsilon_s(t). \tag{5}$$

with δ_c coefficient of longitudinal stiffness of concrete slab given by $\delta_c = 1/(E_c A_c)$.

Applying relationships from (1) to (3), the strains in the top and bottom reference fibres may be written as

$$\varepsilon_{cds}(t) = \frac{4N_c(t)}{E_c(t)A_c} = \frac{4\delta_c}{\delta_a + \delta_c}\varepsilon_s(t)$$

and

$$\varepsilon_{cgs}(t) = \frac{2\delta_c}{\delta_a + \delta_c}\varepsilon_s(t). \tag{6}$$

4. Stresses and strains in concrete slab due to direct load actions

The flexure obviously causes compression in concrete element. For the standard forms of composite beams as shown in Fig. 2, the reduction in the flexural rigidity that can occur through accidental cracking is very small. Possible tensile cracking in bottom fibres of slab can exist if the neutral axis is found to lie in the concrete element. Composite beams with full shear connection are assumed to have full shear interaction so that slip and hence slip strain are ignored. The linear strain profile without any step change is shown in Fig. 2 for this configuration. The steel modulus E_a and the short-term concrete modulus E_c can be considered to be elastic, and for the case of obvious analysis transformed area principles may be adopted. The composite section can be changed into a concrete section by increasing the components of steel elements by the modular ratio of the constituent materials $n = E_a/E_c$. The composite beam is supposed to be subjected to positive bending so that the top fibre of the concrete is in compression. The size and position of all elements are obviously known so that the position of the neutral axis lies at the centroid of the transformed section. Once the neutral axis has been located, the stresses and deformations at top and bottom fibre of the concrete slab may be calculated easily.

$$\varepsilon_{cgp} = \frac{M}{W_{cg}E_c} = \frac{M}{E_c I_{zc}} z_c$$

and

$$\varepsilon_{cdp} = \frac{M}{W_{cd}E_c} = \frac{M}{E_c I_{zc}}(z_c - h_c). \tag{7}$$

With the second moment of area of the effective composite section transformed into the concrete section $I_{zc} = nI_a + I_c + nA_a a_a^2 + A_c a_c^2$, modular ratio of the constituent materials defined as $n = E_a/E_c$, z_c distance between the centroid of the composite section to the extreme fibre of the composite slab in compression, hc thickness of the concrete slab, a_c distance between the centroid of concrete slab to the neutral axis of composite beam, a_a distance between the centroid of steel beam to the neutral axis of composite section, I_c second moment of area of the un-cracked concrete slab, flexural rigidity of the composite beam with full interaction as the sum of flexural rigidities of the individual elements $E_c I_{zc} = B_{zc} = E_a I_a + E_c I_c + E_a A_a a_a^2 + E_c A_c a_c^2$.

5. Control of cracking due to concrete shrinkage

Concrete slab would crack when shrinkage strain satisfies the following criterion:

$$\varepsilon_{cds}(t) = \frac{4N_c(t)}{E_{c(t)}A_c} > \varepsilon_{ct,\lim}. \tag{8}$$

Substituting relations (6), this crack initialization principle, when cracks may first be expected to occur, can be rewritten in the following form:

$$\frac{\varepsilon_{ct,\lim}}{\varepsilon_s(t)} < \frac{4\delta_c}{\delta_a + \delta c}. \tag{9}$$

The limit value of the axial tensile strain of concrete could be calculated from $\varepsilon_{ct,\lim} = \eta_\phi \cdot f_{ctm}/E_{ct}$, with f_{ctm} as mean value of the axial tensile strength of concrete, E_{ct} modulus of elasticity for concrete in tensions, ρ_r percentage of reinforcement related to the area of the tensile cracked zone of the slab cross section, ϕ diameter of reinforcement bar and

$$\eta_\phi = 1 + \frac{0.08\rho_r}{\phi^{1.5}}. \tag{10}$$

With $\eta_c(t) = \frac{f_{ctm}}{E_c\varepsilon_s(t)}$ and $\beta_0 = \frac{1}{\delta} = \frac{\delta_c}{\delta_a}$, the condition for crack initialisation can be expressed in the following more general form

$$\omega = \frac{\eta_c(t)\eta_\phi}{4\beta_0}(1 + \beta_0) < 1 \text{, with } \eta_{c\phi} = \eta_c(t)\eta_\phi \tag{11}$$

6. Cracking produced by concrete shrinkage together with direct load actions

From the relations for determinations of strains ε_{cg} and ε_{cd} is not evident, which of two strains would reach faster the limit strain $\varepsilon_{ct,\lim}$. Crack initialisation depends on the magnitude of shrinkage strains, the dimensions of the elements, the composition of the concrete and on the factor β_0, as well as concrete strength classes and reinforcing steel. The resulting strain diagram shown in Fig.3 indicates that the ultimate or crushing values of extreme strains are at the slab surfaces. The strain profile is thus determined. The top and bottom fibre strains in the slab section, because of the linearity, allow expressing the curvature by the simplified formula

$$\kappa_s = \frac{\varepsilon_{cds} - \varepsilon_{cgs}}{h_c} = \frac{6\delta_c}{(\delta_a + \delta_c)}\varepsilon_s(t), \tag{12}$$

The curvature for resulting strains is similarly

$$\kappa_p = \frac{\varepsilon_{cg} - \varepsilon_{cd}}{h_c} = \frac{M}{B_{zc}}. \tag{13}$$

If $\varepsilon_{cds} < \varepsilon_{ct,\lim}$, cracking in the concrete slab due to shrinkage will not occur. In this case, the strains due to direct load actions to produce slab cracking might reach the magnitude

$$\Delta\varepsilon_{ct}(t) = \varepsilon_{ct,\lim} - \varepsilon_{cds}(t) = \varepsilon_{ct,\lim} - \frac{4\delta_c}{\delta_a + \delta_c}\varepsilon_s(t) \quad (14)$$

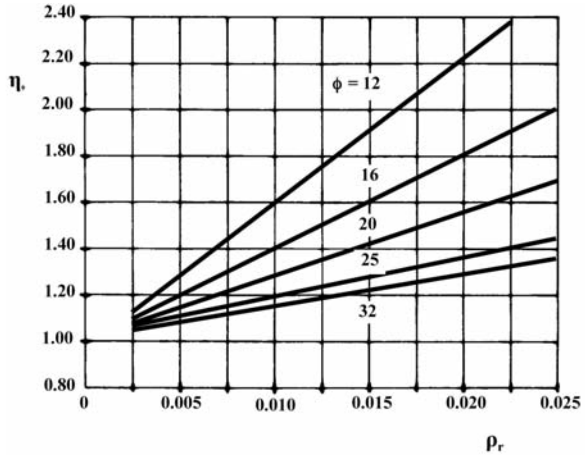

Fig. 4 Percentage ρ_r and diameters ϕ impact on limit strain $\varepsilon_{ct,\lim}$, expressed η_ϕ (10)

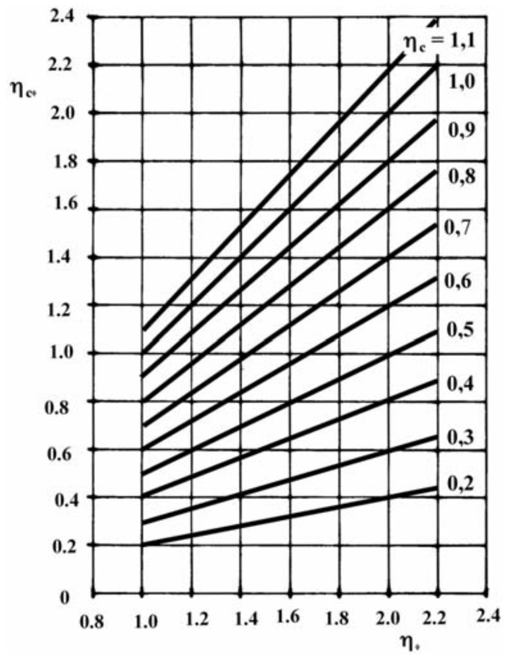

Fig. 5 Factor η_{c_ϕ} variation with diameters ϕ and shrinkage strain ε_s, expressed η_ϕ (11)

This relationship can be written with the ration of both strains $\eta_s = \varepsilon_s(t)/\varepsilon_{ct,\lim}$ as

$$\lambda = \frac{\Delta\varepsilon_{ct}(t)}{\varepsilon_{ct,\lim}} = 1 - \frac{4\beta_0\,\varepsilon_s(t)}{(1+\beta_0)\varepsilon_{ct,\lim}} = 1 - \frac{4\beta_0}{1+\beta_0}\eta_s \quad (15)$$

7. Parametrical study

Influence of the above variable parameters on the concrete slab cracking is shown in Figs.4 – 7. The impact of reinforcement percentage ρ_r and diameters of reinforcement bar ϕ on limit value of the axial tensile strain of concrete $\varepsilon_{ct,\lim}$ is summarised in Fig. 4. The variable parameter is chosen the value η_ϕ.

Relative influence of reinforcement bar diameters and values of the axial tensile strength of concrete is illustrated in Fig. 5. Factors η_ϕ and η_c according to formulae (10) and (11) are taken as variables. Rather large intervals of these factors are studied. Their magnitudes correspond to the actual forms of composite steel and concrete composite structural elements.

The analysis of conventional factor β_0 is shown in Fig. 6. The previously investigated parameters (ϕ, ε_s, f_{ctm}, E_c) are about the concrete slab. The factor β_0 takes into account the steel beam and in the same time the total composite section. When the design value of factor κ in Fig. 6 is superior to one, shrinkage cracks occur without effect of external actions. Cracking starts at the bottom fibre in the shear connection plane. The variation of strains $\Delta\varepsilon_{ct}(t)$ deducing shrinkage for different values of the ratio η_s is shown in Fig. 7. The magnitudes of variable λ could express shrinkage cracking occurring before applications of external load

8. Concluding remarks

The investigations have illustrated possibilities of shrinkage crack occurring in a concrete slab of composite beams even prior to the application of external actions. Except for shrinkage concrete strain, important influence is also presented by percentage of reinforcement, diameter of reinforcement bar, tensile strength of concrete and furthermore the factor β_0, which means a ratio of the stiffness of steel beam and concrete slab. Larger values of concrete strains due to shrinkage arise at the bottom slab surface. Consequently, shrinkage cracking can be initiated primarily at the bottom concrete fibre in the shear connection plane with steel beam flange. For real composite structures, the ratio β_0 can be extremely variable. However, it may be concluded that shrinkage crack would not be initiated, when $\beta_0 < 0.25$. The cracking can be avoided by rational concrete slab reinforcement (percentage of reinforcement ρ_r and bars diameter ϕ). The results of analyses in Figs. 4 to 7 may be useful for control of potential cracking during composite steel and concrete design.

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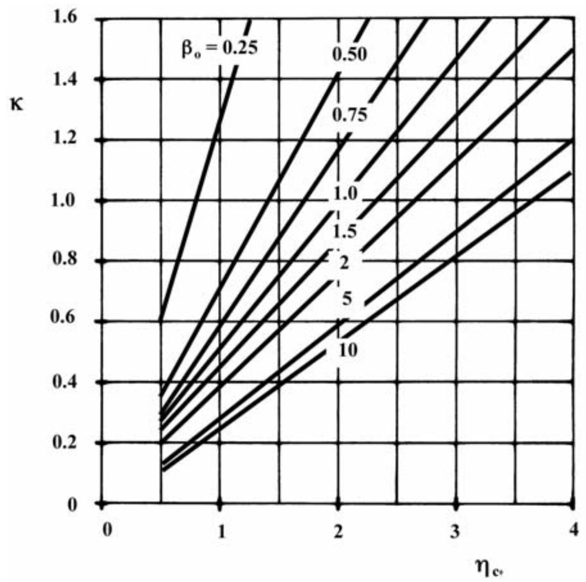

Fig. 6 Impact of factor $\beta_0 = \delta_c/\delta_a$ on the slab curvature κ

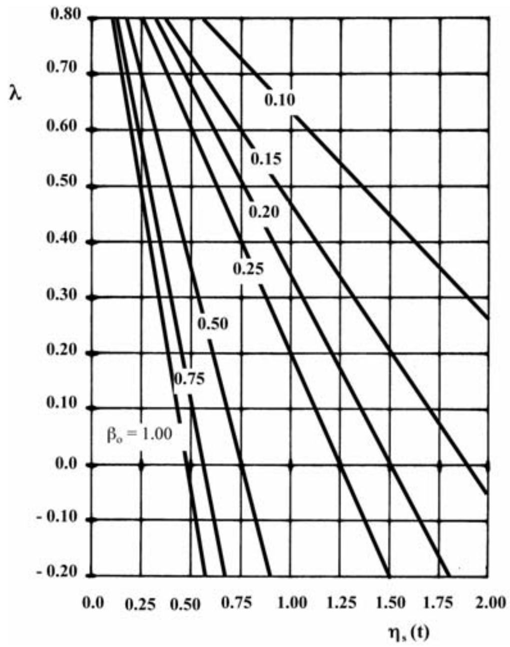

Fig .7 Dependence of strain ratio on λ factor $\beta_0 = \delta_c/\delta_a$

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